

Time : 3 hours

Full Marks : 200

Instructions :

- (1) Answer all questions following the directions.
 (2) The figures in the margin indicate full marks for the questions.

MATHEMATICS

(Marks : 40)

1. Solve the following differential equation : 5

$$x(1-x^2)\frac{dy}{dx} + (2x^2-1)y = x^3$$

2. Find the eigenvalues and eigenvectors of the matrix : 5

$$A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$$

3. If $A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$, then show that

$$A(\text{adj } A) = (\text{adj } A)A = |A|I \quad 5$$

4. If $z(x+y) = x^2 + y^2$, then show that

$$\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right) \quad 5$$

5. Obtain the Fourier series for the function $f(x) = x^2$, $-\pi < x < \pi$. Hence show that

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots = \frac{\pi^2}{6} \quad 5$$

6. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then show that

$$\text{div } \vec{r} = 3 \text{ and } \text{curl } \vec{r} = \vec{0}$$

5

7. Find all the values of $(1+i)^{1/3}$ and obtain their product.

5

8. Assume that on the average one telephone number out of fifteen called between 2 p.m. and 3 p.m. on weekdays is busy. What is the probability that if six randomly selected telephone numbers are called (a) not more than three and (b) at least three of them will be busy?

5

PROCESS CALCULATIONS AND THERMODYNAMICS

(Marks : 30)

9. 1000 kg of mixed acid of composition 40% H_2SO_4 , 45% HNO_3 and 15% H_2O is to be produced by strengthening waste acid of composition 30% H_2SO_4 , 36% HNO_3 and 34% H_2O by weight. Concentrated sulphuric acid of strength 95% and concentrated nitric acid containing 80% are available for this purpose. How many kilograms of spent acid and concentrated acids are to be mixed together?

5

10. Recycling of the fluid stream is an important operation in chemical processing. Explain with a suitable example.

5

11. Propane is burned with excess air to ensure complete combustion. If 55 kg of CO_2 and 15 kg of CO are obtained when propane is completely burned with 500 kg of air, determine the kg of propane burnt and the percentage of excess air required.

5

12. What do you mean by intensive and extensive properties? Explain with examples.

4

13. State and explain the second law of thermodynamics. A 40 kg steel casting ($C_p = 0.5 \text{ kJ/kgK}$) at a temperature of 723.15 K is quenched in 150 kg of oil ($C_p = 2.5 \text{ kJ/kgK}$) at 298.15 K. If there are no heat losses, what is the change in entropy of the steel casting, oil and the both?

2+4=6

14. Show that for an ideal gas

$$\left(\frac{\partial E}{\partial V}\right)_T = 0$$

Or

What do you mean by degrees of freedom? How many degrees of freedom each of the following systems have?

- Liquid water in equilibrium with its vapour
- Liquid water in equilibrium with a mixture of water vapour and nitrogen
- A liquid solution of alcohol in water in equilibrium with its vapour

FLUID MECHANICS AND MECHANICAL OPERATIONS

(Marks : 30)

15. State the different laws of size reduction. It is desired to crush 100 tons/hour of phosphate rock from a feed size where 80% is less than 4 inch to a product where 80% is less than $\frac{1}{8}$ inch. The work index is 2.74.

Calculate the power required.

2+4=6

16. Explain with neat sketch, the working principle of an equipment used for the separation of a gas-solid mixture.

17. How does free settling differ from hindered settling? Show that the maximum spherical particle diameter of density ρ_s obeying Stokes' law range and settling under gravity in a fluid of density ρ and viscosity μ is given by

$$D_{p_{\max}} = \left[\frac{36\mu^2}{\rho(\rho_s - \rho)g} \right]^{1/3}$$

Or

Explain with neat sketch, the concept of hydrodynamic boundary layer. Why is it important in fluid mechanics?