500029

F/I/R/M EXAMS
2020

MATHEMATICS

Time : 3 hours] [Full Marks : 100

Notes : *(i)* Answer any **five** questions.

(ii) The figures in the right-hand margin indicate full marks for the questions.

1. Answer *all* questions : 4×5=20

(a) If $U = \tan^{-1}\dfrac{x^3+y^3}{x-y}$, show that $x\dfrac{\partial U}{\partial x} + y\dfrac{\partial U}{\partial y} = \sin 2U$.

(b) Evaluate $\Delta^n(e^{ax})$.

(c) Let $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 2 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$ be two permutation of order 4. Show that $\alpha\beta \neq \beta\alpha$.

(d) If α, β, γ are the roots of the equation $x^3 + px + q = 0$, find the value of

$$\frac{1}{\alpha+\beta} + \frac{1}{\beta+\gamma} + \frac{1}{\gamma+\alpha}$$

(e) Evaluate $\lim\limits_{x\to 0}\left(\dfrac{1}{x^2} - \operatorname{cosec}^2 x\right)$.

2. Write short notes on the following : 4×5=20

(a) Compact set

(b) Bayes' theorem

(c) Linear Programming Problem (LPP)

(d) Riemann integrable function

(e) Analytic function

3. Answer *all* questions : 5×4=20

(a) Show that $W = \{(x, y, z) | x - 3y + 4z = 0\}$ is a subspace of $\mathbb{R}^3$ over the field of real number.

(b) Show that the semi-vertical angle of a cone of maximum volume and given slant height, is $\tan^{-1}\sqrt{2}$.

(c) Find the area common to the circle $x^2 + y^2 = 4$ and ellipse $x^2 + 4y^2 = 9$.

(d) Prove that the set of all nth roots of unity forms a finite Abelian group of order n with respect to multiplication.

4. Answer *all* questions : 5×4=20

(a) Solve $(1 + y^2)dx = (\tan^{-1} y - x)dy$.

(b) Show that the function $u(x, y) = 2x + y^3 - 3x^2 y$ is harmonic. Find its conjugate harmonic function $v(x, y)$ and the corresponding analytic function $f(z)$.

(c) A student obtained the mean and standard deviation of 100 observations are 40 and 5 respectively. It was later discovered that he had wrongly copied down an observation as 50 instead of 40. Calculate the correct mean and standard deviation.

(d) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Simpson's $\frac{3}{8}$ rule.

5. Answer *all* questions : 5×4=20

(a) If g.c.d. of 7 and n is 1, then prove that $n^6 - 1$ is divisible by 7.

(b) By using Newton's forward formula, find the value of $f(x)$ at $x = 2 \cdot 1$ from following data :

x	:	0	2	4	6	8
$f(x)$	:	−1	13	43	89	151

(c) State and prove Lagrange's theorem.

(d) Find the value of λ so that the equation $2x^2 + xy - y^2 - 11x - 5y + \lambda = 0$ may represent a pair of straight lines.

6. (a) Find the equations to the pair of lines through the origin and perpendicular to the pair of lines $ax^2 + 2hxy + by^2 = 0$. 5

   (b) Find the equation of sphere passing through the four points (1, 2, 3), (0, −2, 4), (4, −4, 2) and (3, 1, 4). 5

   (c) Use Green's theorem to evaluate $\int_C [(y - \sin x)dx + \cos x\, dy]$, where C is the triangle enclosed by the lines $y = 0$, $x = \frac{\pi}{2}$ and $y = \frac{2x}{\pi}$. 10

7. (a) Use elementary row transformation to compute the inverse of the matrix

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$ 5

   (b) If f and g are two Riemann integrable functions on $[a, b]$, then show that $f + g$ is also Riemann integrable on $[a, b]$. 5

   (c) Find the maxima and minima of the function

$$f(x, y) = x^3 + y^3 - 3x - 12y + 20$$ 10

8. (a) Evaluate the integral

$$\oint_C \frac{z^2 + 1}{z(2z - 1)} dz, \; C : |z| = 1$$ 5

   (b) A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30 g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires at least 240 units of calcium, at least 460 units of iron and at most 300 units of cholesterol. How many packets of each food should be used to minimise the amount of vitamin A in the diet? What is the minimum amount of vitamin A? 10+5=15