CCE(M)/MTH–II/10/2024

MATHEMATICS

(PAPER—II)

Time : 3 hours] [Full Marks : 250

Please read each of the following instructions carefully before attempting the questions :

(i) There are **eight** questions divided in **two Sections**.

(ii) Candidate has to attempt **five** questions in all.

(iii) Question Nos. **1** and **5** are compulsory and out of the remaining, **three** are to be attempted choosing at least **one** question from each Section.

(iv) The number of marks carried by a question/part is indicated against it.

(v) Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

SECTION—A

1. Answer *any* **five** of the following questions : 10×5=50

(a) Discuss the convergence and absolute convergence of the series $x-\dfrac{x^3}{3}+\dfrac{x^5}{5}-\ldots$, x being real.

(b) Show that $z=a$ is an isolated essential singularity of the function $\dfrac{e^{c/(z-a)}}{e^{z/a}-1}$.

(c) Find a positive root of the equation $x^3=2x+5$ by the method of false position correct to three places of decimal.

(d) Solve $\dfrac{\partial^2 z}{\partial x^2}+3\dfrac{\partial^2 z}{\partial x \partial y}+2\dfrac{\partial^2 z}{\partial y^2}=x+y$.

(e) Find Taylor's series valid in the neighbourhood of the point $z = i$ for the function

$$f(z) = \frac{2z^3 + 1}{z^2 + z}.$$

(f) Show that the sequence $\langle x^{n-1}(1-x) \rangle$ is uniformly convergent on $[0,1]$.

(g) Prove that $\dfrac{\pi^2}{9} \le \displaystyle\int_{\pi/6}^{\pi/2} \frac{x}{\sin x}\, dx \le \frac{2\pi^2}{9}$.

2. Answer the following questions :

(a) Find all the homomorphisms from $\dfrac{\mathbb{Z}}{4\mathbb{Z}}$ to $\dfrac{\mathbb{Z}}{6\mathbb{Z}}$. 20

(b) Show that $\mathbb{Z}[\sqrt{2}] = \{a + \sqrt{2}\, b : a, b \in \mathbb{Z}\}$ is a Euclidean domain. 15

(c) By using the method of residues, show that $\displaystyle\int_0^{\pi} \frac{d\theta}{17 - 8\cos\theta} = \frac{\pi}{15}$. 15

3. Answer the following questions :

(a) Show that the sequence $\langle \tan^{-1} nx \rangle$, $x \ge 0$, is uniformly convergent on any interval $[a, b]$, $a > 0$ but is only pointwise convergent on $[0, b]$. 20

(b) Prove that the function f defined by $f(x) = \sin\dfrac{1}{x}\ \forall x > 0$ is continuous but not uniformly continuous on $\mathbb{R}^+$. 15

(c) Prove that the intersection of all conjugates of a subgroup H in G is a normal subgroup of G. 15

4. Answer the following questions :

(a) Use simplex method to solve the following linear programming problem :
Maximize $Z = 3x_1 + 5x_2 + 4x_3$
Subject to constraints :
(i) $2x_1 + 3x_2 \le 8$, (ii) $2x_2 + 5x_3 \le 10$, (iii) $3x_1 + 2x_2 + 4x_3 \le 15$, where $x_1, x_2, x_3 \ge 0$.
20

(b) By using the Lagrange's interpolation formula fit a polynomial to the following data :

x	-1	0	2	3
y	-8	3	1	12

and hence find $y(x=1)$. 15

(c) Find the particular integral of the following partial differential equation :

$$(D^2 - DD' - 2D'^2)z = (2x^2 + xy - y^2)\sin xy - \cos xy,$$

where $D \equiv \frac{\partial}{\partial x}$, $D^2 \equiv \frac{\partial^2}{\partial x^2}$, $D' \equiv \frac{\partial}{\partial y}$ and $D'^2 \equiv \frac{\partial^2}{\partial y^2}$. 15

SECTION—B

5. Answer *any* **five** of the following questions : 10×5=50

(a) If in a group, $a^5 = e$, $aba^{-1} = b^2$ for $a, b \in G$ and identity element e, then show that $o(b) = 31$.

(b) By using Cauchy's appropriate theorem of limits, prove that the sequence $\left\{\left(\frac{(3n)!}{(n!)^3}\right)^{\frac{1}{n}}\right\}$ is convergent.

(c) Form a partial differential equation by eliminating the arbitrary function $\phi(x+y+z,\ x^2+y^2-z^2) = 0$. What is the order of this partial differential equation?

(d) Let $f(z) = u + iv$ be an analytic function. Find $f(z)$ if $u - v = \frac{\cos x + \sin x - e^{-y}}{2\cos x - e^{y} - e^{-y}}$, where $f\left(\frac{\pi}{2}\right) = 0$.

(e) Show that the integral $\int_0^{\frac{\pi}{2}} \sin x \log(\sin x)\, dx$ is convergent and its value is $\log\left(\frac{2}{e}\right)$.

(f) By using $\in - \delta$ definition, show that the function $f(x,y) = x^2 + 3y$ is continuous at (1,2).

(g) Evaluate $\oint_C \frac{z+1}{z^4 - 2z^3} dz$, where C is the circle $|z| = \frac{1}{2}$ by using residue theorem.

6. Answer the following questions :

(a) Solve the following system of linear equations $8x - 3y + 2z = 20$, $4x + 11y - z = 33$, $6x + 3y + 12z = 35$ by using Gauss-Seidel method correct to three places of decimal. 20

(b) Apply Gauss-Jordan method to find the solution of the following system of equations $10x + y + z = 12$, $2x + 10y + z = 13$, $x + y + 5z = 7$. 15

(c) If $\frac{dy}{dx} = \frac{2xy + e^x}{x^2 + xe^x}$, $y(1) = 0$, find the value of y at $x = 1.2$ by using Runge-Kutta method of fourth order. Take the step size as $h = 0.2$. 15

7. Answer the following questions :

(a) A company has factories at D, E and F which supply to warehouses at W, X and Y. Weekly factory capacities are 200, 160 and 90 units, respectively. Weekly warehouse requirements are 180, 120 and 150 units, respectively. Unit shipping cost (in Rupees) are as follows :

Warehouse

		W	X	Y	Supply
Factory	D	16	20	12	200
	E	14	8	18	160
	F	26	24	16	90
	Demand	180	120	150	

Find the initial basic feasible solution by North-West corner method.

Determine also the optimal distribution for this company to minimize total shipping cost by using MODI method. 20

(b) An air force is experimenting with three types of bombs P, Q and R in which three kinds of explosives are viz A, B and C will be used. Taking the various factors into account, it has been decided to use the maximum 600 kg of explosive A, at least 480 kg of explosive B and exactly 540 kg of explosive C. Bomb P requires 3, 2, 2 kg, bomb Q requires 1, 4, 3 kg and bomb R requires 4, 2, 3 kg of explosive A, B and C respectively. Bomb P is estimated to size the equivalent of a 2 ton explosion, bomb Q, a 3 ton explosion and bomb R, a 4 ton explosion respectively. Under what production schedule can the air force make the biggest bang? 15

(c) Consider the following transportation problem :

Destination

		1	2	3	4	Supply
Origin	1	20	22	17	4	120
	2	24	37	9	7	70
	3	32	37	20	15	50
	Demand	60	40	30	110	

Find the initial basic feasible solution of the above problem by using Vogel's approximation method. 15

8. Answer the following questions :

(a) Show that the moment of inertia of an elliptic area of mass M and equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ about a diameter parallel to the axis of x is $-(aM\Delta)/\{4(ab-h^2)^2\}$, where $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$. 20

(b) Consider the velocity field given by $\vec{q} = \{1 + At\}\hat{i} + \{x\}\hat{j}$. Find the equation of the streamline at $t = t_0$ passing through the point (x_0, y_0). Also obtain the equation of the path line of a fluid element which comes to (x_0, y_0) at $t = t_0$. Show that, if $A = 0$, the streamline and path line coincide. 15

(c) Determine the constants l, m and n in order that the velocity component

$$\vec{q} = \left\{\frac{x + lr}{r(x+r)}\right\}\hat{i} + \left\{\frac{y + mr}{r(x+r)}\right\}\hat{j} + \left\{\frac{z + nr}{r(x+r)}\right\}\hat{k}$$

where $r^2 = x^2 + y^2 + z^2$ may satisfy the equation of continuity for a liquid. 15

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SPACE FOR ROUGH WORK

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