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Invigilator's signature

Question Booklet No. 690031

2018

PGT — PAPER - I : MATHEMATICS

Time : 2 Hours **Maximum Marks : 100**

ROLL NO. ☐☐☐☐☐☐

INSTRUCTIONS FOR CANDIDATES

1. This Question Booklet contains 50 optional questions. Each question comprises four responses (answers). You will select ONLY ONE response which you consider the best and darken the bubble on the OMR RESPONSE SHEET.
2. DO NOT write your Name or anything else except Roll No. and the actual answers to the question, anywhere on the OMR RESPONSE SHEET.
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1. If a function $f(x)$ is periodic with period p, then the function $f(ax+b)$ where $a, b \in R$, $(a \neq 0)$, is also a periodic function with period

(A) pa

(B) $p + a$

(C) $\frac{p}{|a|}$

(D) $p + |a|$

2. The limit

$$\lim_{x \to 2}\left[\frac{(10-x)^{\frac{1}{3}}-2}{x-2}\right]$$

evaluates to

(A) $\frac{1}{2}$

(B) $\frac{1}{10}$

(C) $\frac{1}{12}$

(D) $-\frac{1}{12}$

3. If $f(x) = 2x^p + q$, $\forall p, q \in I$, $f(1) = 12$ and $f(3) = 64$, then $f(4)$ is

(A) 138

(B) 128

(C) 148

(D) 158

4. The range of the function

$$f(x) = \sqrt{x-4} + \sqrt{6-x} \quad (x \in I)$$

is

(A) $\left[-\frac{3}{2}, -1\right]$

(B) $\left[\sqrt{2}, 2\right]$

(C) $\left[\frac{1}{3}, 1\right]$

(D) [0, 1]

5. The locus of the point z satisfying the condition $\arg\left[\frac{z-1}{z+1}\right] = \frac{\pi}{3}$ is

(A) an ellipse

(B) a circle

(C) a parabola

(D) None of the above

6. If 1, ω, ω^2 are the three cube roots of unity, then the roots of the equation $(x-1)^3 - 8 = 0$ are

(A) 3, $1-2\omega$, $1-2\omega^2$

(B) 3, 2ω, $2\omega^2$

(C) 3, 3ω, $3\omega^2$

(D) 3, $1+2\omega$, $1+2\omega^2$

7. In an office, every employee posts one greeting card to every other on a New Year. If the postman delivers a total of 272 such cards, the number of employee in the office is

(A) 15

(B) 17

(C) 20

(D) 22

8. In how many different ways three persons A, B, C having 6, 7 and 8 one rupee coins respectively can donate Rs. 10 collectively?

(A) ${}^{12}C_2 - {}^{5}C_2 - {}^{4}C_2 - {}^{3}C_1$

(B) ${}^{8}C_2 + {}^{7}C_2 + {}^{6}C_2$

(C) ${}^{12}C_2 \times {}^{5}C_2 \times {}^{4}C_2 \times {}^{3}C_1$

(D) None of the above

9. If $\sin x = \frac{1}{\sqrt{2}}$, then cos 3x equals

(A) $-\frac{1}{\sqrt{2}}$

(B) $\frac{1}{\sqrt{3}}$

(C) $\frac{1}{2}$

(D) 0

10. If $\tan\left(\frac{\pi}{6} - \theta\right) \tan\left(\frac{\pi}{6} + \theta\right) = \infty$, then the value of $\tan\theta$ is

(A) 0

(B) 1

(C) $\sqrt{3}$

(D) ∞

11. Let $f(x) = 1 - \frac{1}{x}$ for all $x, y \in R$. Then $f(f(f(x)))$ is equal to

(A) $1 - x$

(B) $\frac{1}{x}$

(C) x

(D) $\frac{x}{2}$

12. If $f'(x) = x^2 - x + 1$ and $y = f(x^2)$, then $\frac{dy}{dx}$ at $x = -1$ is

(A) -2

(B) -1

(C) 0

(D) 1

13. If $\int_0^3 f(x)\,dx = 2$, then the integration $\int_0^1 f(x)\,dx + \int_0^1 f(x+1)\,dx + \int_0^1 f(x+2)\,dx$ evaluates to

(A) 0

(B) 2

(C) 5

(D) 6

14. The solution of the differential equation

$$\frac{dy}{dx}=\frac{y}{x}+\frac{\varphi\left(\frac{y}{x}\right)}{\varphi'\left(\frac{y}{x}\right)}$$ is, (k is a constant)

(A) $\varphi\left(\frac{y}{x}\right)=\frac{k}{x}$

(B) $\varphi\left(\frac{y}{x}\right)=\frac{k}{y}$

(C) $\varphi\left(\frac{y}{x}\right)=kx$

(D) $\varphi\left(\frac{y}{x}\right)=ky$

15. The differential equation for a simple harmonic motion with the angular velocity $\frac{1}{\omega}$ is

(A) $\frac{d^2x}{dt^2}-\frac{x}{\omega^2}=0$

(B) $\frac{d^2x}{dt^2}-\omega^2x=0$

(C) $\frac{d^2x}{dt^2}+\omega^2x=0$

(D) $\frac{d^2x}{dt^2}+\frac{x}{\omega^2}=0$

16. The point of intersection and the angle between the straight lines $2x+3y+2=0$ and $y-x=4$ are respectively

(A) $(2,-2)$; $\tan^{-1}5$

(B) $\left(-\frac{14}{5},\frac{6}{5}\right)$; $\tan^{-1}5$

(C) $(2, 2)$; $\tan^{-1}3$

(D) $(2,-2)$; $\tan^{-1}3$

17. The length of the perpendicular from the point $(7, 6)$ to the straight line $2x-2y+1=0$ is

(A) 2

(B) 0

(C) 5

(D) $\frac{3}{2\sqrt{2}}$

18. Find unit vector in the direction of vector $\vec{a}=2\hat{i}+3\hat{j}+\hat{k}$

(A) $\hat{a}=\frac{1}{\sqrt{14}}\left(2\hat{i}+3\hat{j}+\hat{k}\right)$

(B) $\hat{a}=\frac{1}{\sqrt{5}}\left(2\hat{i}-\hat{j}+\hat{k}\right)$

(C) $\hat{a}=\frac{1}{\sqrt{14}}\left(\hat{i}+3\hat{j}-\hat{k}\right)$

(D) $\hat{a}=\frac{1}{\sqrt{14}}\left(\hat{i}+3\hat{j}+\hat{k}\right)$

19. The vector $\vec{A}\times\left(\vec{A}\times\vec{B}\right)$ lies in the plane

(A) containing $\vec{A}$ and $\left(\vec{A}\times\vec{B}\right)$

(B) containing $\vec{B}$ and $\left(\vec{A}\times\vec{B}\right)$

(C) perpendicular to both $\vec{A}$ and $\vec{B}$

(D) containing both $\vec{A}$ and $\vec{B}$

20. 1 terrabyte equals

(A) 2^{10} bytes

(B) 2^{20} bytes

(C) 2^{30} bytes

(D) 2^{40} bytes

21. *Mathematica* is

(A) an operating system

(B) a machine level language

(C) an application software

(D) a database management software

22. The statement that 'Every matrix satisfies its own characteristic equation', constitutes

(A) Cayley's theorem

(B) Rank-Nullity theorem

(C) Hamilton's theorem

(D) Cayley-Hamilton theorem

23. The series $\frac{1}{2}+\frac{4}{9}+\frac{9}{28}+\ldots+\frac{n^2}{n^3+1}+\ldots$ is

(A) convergent

(B) divergent

(C) oscillatory

(D) semi-convergent

24. If α, β, γ are the roots of $x^3-7x^2+36=0$, then the value of $\alpha\beta+\beta\gamma+\gamma\alpha$ equals

(A) 0

(B) 1

(C) 7

(D) -36

25. For any real numbers a_i, b_i $(i = 1, 2, \ldots, n)$, the expression $(a_1b_1+a_2b_2+\ldots+a_nb_n)^2 \leq$

$$\left(a_1^2+a_2^2+\ldots+a_n^2\right)\left(b_1^2+b_2^2+\ldots+b_n^2\right)$$

constitutes

(A) Schwarz inequality

(B) Cauchy inequality

(C) Cauchy-Schwarz inequality

(D) Arithmetic-Geometric inequality

26. A fair coin is tossed 4 times. The probability of getting at least 2 heads is

(A) 1/2

(B) 3/8

(C) 5/8

(D) 11/16

27. The quantity which gives the measure of the shape of the probability distribution curve is

(A) dispersion

(B) skewness

(C) mean deviation

(D) standard deviation

28. The theorem which relates the contour integral to the surface integral associated with a vector field is

(A) Gauss theorem

(B) Helmholtz theorem

(C) Stokes' theorem

(D) Maxwell's theorem

29. The relation 'less than' defined in the set of real numbers $\boldsymbol{R}$ is

(A) reflexive only

(B) symmetric only

(C) reflexive and transitive

(D) antisymmetric and transitive

30. A ring ($\boldsymbol{R}$, +, .) with all its elements idempotent is known as the

(A) Boolean ring

(B) p-ring

(C) integral domain

(D) ideal

31. Let $W_1 = \{(x, y, z) \in R^3 : x = 2y\}$, $W_2 = \{(x, y, z) \in R^3 : xy = 0\}$. Then

(A) $W_1 \cap W_2$ is a subspace of R^3

(B) W_1 is a subspace of R^3

(C) W_2 is a subspace of R^3

(D) all three are true

32. For the vector spaces $V(F)$ and $U(F)$, the linear map $f: V \to U$ is the vector space isomorphism if f is

(A) many-one into

(B) one-one into

(C) one-one onto

(D) many-one onto

33. For non-identical numbers, the relationship among the Arithmetic mean (AM), Geometric mean (GM) and the Harmonic mean (HM) is

(A) AM $\leq$ GM $\leq$ HM

(B) GM $\leq$ AM $\leq$ HM

(C) HM $\leq$ AM $\leq$ GM

(D) AM $\geq$ GM $\geq$ HM

34. The differential equation $M(x, y)\,dx + N(x, y)\,dy = 0$ is exact when

(A) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

(B) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$

(C) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial y}$

(D) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial x}$

35. For a positive integer k, if the eigenvalues of the matrix $A_{n\times n}$ are $\lambda_1, \lambda_2, \ldots, \lambda_n$, then the eigenvalues of $A^k_{n\times n}$ are given by

(A) $\lambda_1 + k, \lambda_2 + k, \ldots, \lambda n + k$

(B) $k\lambda_1, k\lambda_2, \ldots, k\lambda_n$

(C) $\lambda_1/k, \lambda_2/k, \ldots, \lambda_n/k$

(D) $\lambda_1^k, \lambda_2^k, \ldots, \lambda_n^k$

36. For a matrix $A_{n\times n}$, if it is always possible to find n linearly independent and mutually orthogonal eigenvectors, the matrix $A_{n\times n}$ is

(A) Hermitian

(B) real symmetric

(C) anti-Hermitian

(D) upper triangular

37. The product of the eigenvalues of a given matrix $\boldsymbol{A}$ is equal to the

(A) trace of $\boldsymbol{A}$

(B) determinant of $\boldsymbol{A}$

(C) trace of $\boldsymbol{A}^{-1}$

(D) determinant of $\boldsymbol{A}^{-1}$

38. If $x+\frac{1}{x}=2\cos\theta$, then $x^3+\frac{1}{x^3}$ is given by

(A) $2\cos 3\theta$

(B) $2\sin 3\theta$

(C) $3\cos 2\theta$

(D) $3\sin 2\theta$

39. The sum $\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}$ equals

(A) e^x

(B) $\log x$

(C) $\sin x$

(D) $\cos x$

40. The principal value of $\log(1+i)$ is

(A) $i+\log_e\sqrt{2}$

(B) $\frac{i\pi}{4}+\log_e\sqrt{2}$

(C) $i\sqrt{2}+\log_e\sqrt{2}$

(D) $2i+\log_e\sqrt{2}$

41. The origin of the parabola $y^2=4ax$ lies at the vertex. If the origin is shifted to its focus, the new equation of the parabola is

(A) $y^2=4a(x+a)$

(B) $y^2=4a(x-a)$

(C) $(y-a)^2=4ax$

(D) $(y+a)^2=4ax$

42. The locus of the point from which the sum of the distances to two fixed distinct points is a constant, defines

(A) a circle

(B) a hyperbola

(C) an ellipse

(D) a parabola

43. The equation $y^2=ax^2+b$, $a\neq 1$, $b\neq 0$ represents a/an

(A) circle

(B) hyperbola

(C) ellipse

(D) parabola

44. Find the value of θ, where $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely real.

(A) $n\pi$, $n\in I$

(B) $n\pi/2$, $n\in I$

(C) 0

(D) $n\pi/3$, $n\in I$

45. The range of x for the Gregory series $\tan^{-1}x=x-\frac{x^3}{3}+\frac{x^5}{5}-\ldots\infty$ to be valid, is

(A) $-\frac{\pi}{4}\leq x\leq\frac{\pi}{4}$

(B) $-\frac{\pi}{2}\leq x\leq\frac{\pi}{2}$

(C) $0\leq x\leq\pi$

(D) $-1\leq x\leq 1$

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46. The statement that every finite group is isomorphic to a permutation group is the content of the

(A) Bartrand's theorem

(B) Lagrange's theorem

(C) Cauchy thoerem

(D) Cayley's theorem

47. The degree of the differential equation $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2+\sin y+y^3=0$ is

(A) 1

(B) 2

(C) 3

(D) Degree not defined

48. $\lim_{x\to 0}\left(\frac{\sin^{-1}x}{x}\right)^2$ evaluates to

(A) 1

(B) 0

(C) $\frac{1}{2}$

(D) –1

49. Simplex algorithm is the term related to

(A) computer language

(B) Fourier transform

(C) linear programming

(D) finite element method

50. The maximum value of $z = 4x + y$ subject to the constraints $x + y \le 50$, $3x + y \le 90$, $x \ge 0$, $y \ge 0$ will be

(A) 80

(B) 90

(C) 120

(D) 140

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