SEAL

700051

MATHEMATICS

Paper-II

Time: 3 Hours | Full Marks: 100

Insturctions : *(1) Answer any **five** questions.*

(2) The figures in the right-hand margin indicate full marks for the questions.

1. (*a*) Define a Relation with an example. Describe at least two properties of Relations. Calculate the total number of distinct Relations from set A with m number of distinct elements, to set B with n number of distinct elements. 10

(*b*) Distinguish between the following : $2\times5=10$

(i) Domain and Range of a relation

(ii) Reflexive and Symmetric relations

(iii) Homomorphism and Isomorphism in Groups

(iv) Rings and Fields

(v) Polynomial function and Rational function

2. Write short notes on the following : $4\times5=20$

(*a*) Cauchy-Schwarz inequality

(*b*) Permutation group

(*c*) Riemann integral

(*d*) Normal distribution

(*e*) Vector space

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3. Answer the following : $4\times5=20$

(*a*) Evaluate the limit

$$\lim_{x\to a}\frac{x^m-a^m}{x-a}$$

for positive integer values of m.

(*b*) Find the derivative of the function $f(x)=\sin x$ from the first principle.

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(*c*) Integrate :

$$\int \frac{x^2+4}{x^2+2x+3}\,dx$$

(*d*) Solve the ordinary differential equation $y = px + p - p^2$ where $p = \frac{dy}{dx}$.

(*e*) Differentiate

$$\frac{d^5}{dx^5}\left[e^{ax}\cos(bx+c)\right]$$

where a, b, c are non-zero constants.

4. (*a*) Consider the general second degree equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ and derive the condition under which the equation represents a pair of straight lines. 10

(*b*) Deduce the equation of the tangent plane at a point $P(x',y')$ on the surface of the sphere given by the equation $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$. 10

5. Define infinite series and their convergences. Give the detail of the steps involved in D' Alembert's test to check the convergence of infinite series. Check if the series

$$\frac{x}{1.2}+\frac{x^2}{2.3}+\frac{x^3}{3.4}\text{.........}$$

is convergent. 4+8+8=20

6. State and prove the theorem of total probability and the Bayes' theorem. There are 3 boxes, labelled Bl, B2 and B3 respectively, containing some apples and mangoes. Bl contains 4 apples and 2 mangoes, B2 contains 2 apples and 4 mangoes, and B3 contains 1 apple and 1 mango. Now one fruit is drawn from a random selection of any of the boxes and it is found to be a mango. Find the probability that the mango so drawn belongs to box B3.

5+5+10=20

7. What are eigenvalues and eigenvectors? Determine all the eigenvalues and the eigenvectors of the matrix

$$A = \begin{pmatrix} 1 & 2 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix}$$

4+6+10=20

8. (*a*) Solve the following set of simultaneous equations by Cramer's rule : 8

$$2x + y - z = -4$$

$$-2x + 4y + 3z = 9$$

$$7x - 5y - 2z = 2$$

(*b*) Find out the square root of the complex number

$$\frac{-1+i\sqrt{3}}{2}$$ 6

(*c*) Find the divergence and the curl of the following vector function defined in Cartesian coordinates : 6

$\vec{A}(x, y, z) = xz^3\hat{i} - x^2yz\hat{j} + 2yz^4\hat{k}$ evaluated at the point (1, –1, 1).

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