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GOVERNMENT OF ARUNACHAL PRADESH
DEPARTMENT OF PLANNING
ITANAGAR.

SYLLABUS FOR DIRECT RECUITMENT OF RESEARCH OFFICER

MATHEMATICS – PAPER –I (Pg. No. 2-7) & II (Pg. No. 8-21)

MATHEMATICS

Paper-I

NUMBER THEORY

UNIT I: Divisibility, Division algorithm, Primes, Congruence's, Linear congruence, Chinese Remainder theorem, Fermat`s theorem, Wilson`s theorem, Euler phi function , Euler`s theorem.

UNIT II: Quadratic Congruence: Congruence Modulo powers of prime : power residues: primitive roots and their existence : quadratic residues ;Legendre symbol: Gauss`s lemma about Legendre symbol : quadratic reciprocity law: Jacobi symbol.

UNIT III: Greatest integer function, Arithmetic functions; Multiplicative Arithmetic function; Mobius inversion formula; convolution of arithmetic functions; group properties of arithmetic functions: recurrence functions: Fibonacci numbers and their properties.

UNIT IV: Diophantine equations: solutions of ax + by = c..... Properties of Pythagorean triples: sum of two and three squares; assorted examples of Diophantine equations.

REAL ANALYSIS

UNIT I: The Riemann integral: definitions and condition of integrabitility. Darboux's theorem. integrability of the sum and difference of integrable functions, Fundamental theorem of Calculus. Mean value theorems. Definition. Existence of R.S. integral. properties of R.S – integral.

UNIT II: Uniform convergence. sequence and series of functions. point-wise and uniform convergence . Cauchy`s criterion for uniform convergence of a series, uniform convergence and continuity, integration and differentiation, Weirstrass`s approximation theorem. Power Series.

UNIT III: Lebsegue exterior measure, Lebesgue of sets, theorems on measurable sets. Definiton of measurable functions and properties of measurable functions, simple functions.

UNIT IV: Lebesgue integral of bounded function, definition & theorem involving Lebesgue integral, Relationship of Lebsegue and Riemann integral. Fatou`s Lemma, monotone convergence theorem, the general Lebesgue intergral, Lebesgue convergence theorem.

ALGEBRA

UNIT I: Review of groups. Class equations of finite groups. Sylow`s theorems. Direct of finite numbers group. Decomposable groups. Normal and Subnormal series of groups. Composition series. Jordan Holder theorem. Solvable groups.

UNIT II: Ideals. Principal and Prime ideals. Integral domain and quotients of an integral domain. Divisibility in Commutative rings. PID, UFD and their properties. Eisenstein`s irreducibility criterion.

UNIT III: Field theory. Extension fields. Algebraic and Transcendental numbers. Splitting field. Perfect fields. Existence of finite fields.

UNIT IV: Galois group, Fundamental theorem of Galois theory, Galois group of a cubic equation, Galois group of the general equation. Solvability of equations by radicals, Galois group of a quadratic equation. Constructability of regular polygons.

MECHANICS

UNIT I: Forces in three dimensions, general condition of equilibrium, central axis, pitch, wrench. Resultant of two Wrenches.

UNIT II: Central orbit, Kepler`s laws related to two body problems. Planetary motion.

UNIT III: Motion in 2D: D Alembert`s Principle, motion of a rigid body under impulsive forces. 2D motion of a rigid body about a fixed point and about a fixed axis; euler's Geometrical and dynamical equations. Motion under no external forces.

UNIT IV: Motion in three dimensions, Velocity and acceleration in cylindrical and spherical polar coordinates. Motion on cylindrical; spherical surfaces. Simple Applications.

PROGRAMMING IN C

UNIT I: Computer Organisation: CPU, Main Memory. Input/Output Devices, Classification of Software: Operating System. Introduction to Concept of Computer Networking: LAN, WAN, Ethernet, Intranet, Internet, Network Topology, and Problem Solving Technique: Flow-chart, Decision Table.

UNIT II: Basic feature of C programming languages: data type and variables, identifier, expression and operations. Control statements: do statement, while statement, for statement, nested loop, If-then-else statement, Switch statement, go to, and break and continue statement.

UNIT III: Arrays and pointer in C. structure and union in C, storage mechanism for arrays and pointer in C, string and file handling in C programming language.

UNIT IV: Use of function in C language. Parameter passing mechanism in C, Recursive function. Library function in C. Concept of preprocessing and preprocessor directives. Concept and use of macro.

COMPLEX ANALYISIS

UNIT I: Complex functions. limits and continuity. Differentiable functions of complex variable. Analytic functions. Cauchy Reimann equations. Harmonic functions.

UNIT II: Cauchy-Goursat theorem. Cauchy`s integral formula. Higher order derivatives. Cauchy`s inequalities, Morera`s and Liovelle`s theorem, Gauss`s Mean value theorem, Maximum Modulus theorem.

UNIT III: Taylor`s and Laurent`s theorem, Zero and Singularity of an analytic function. Meromorphic function. The Argument principle. Rouche`s theorem.

UNIT IV: Residue, Calculation of Residues, Cauchy`s Residue theorem, Evaluation of definite integrals, Special theorems used in evaluating integrals, Mittag-Leffer`s theorem.

UNIT V: Isogonal and Conformal transformations: Bilinear transformations. Geometrical inversion. Co-axial circles. Invariance of the Cross-Ratio. Fixed points of a bilinear transformation, some special bilinear transformations e.g. real axis on itself, unit circle on itself, axis on the unit circle.

LINEAR ALGEBRA

UNIT I: Definitons of vector space and subspace. Bases and dimensions. Quotient spaces. Linear transformations, the rank and nullify theorem, inverses of linear transformations. Representation of linear transformations by matrices. Change of Bases. Transpose of linear transformation. Linear operators.

UNIT II: Eigen values and eigen vector, Cayley-Hamilton theorem. Real quadratic form, Matrix of a quadratic form, Criterion positive definiteness, trace and transpose. Characteristic and minimal polynomial of a linear operator. Canonical forms Hermitian, unitary and normal transformation. Invariant subspace. Nilpotent transformation. Invariants of a nilpotent transformation. Direct sum decomposition and primary decomposition theorem .

UNIT III: Dual basis, dual spaces, second dual spaces, annihilators, cyclic subspaces, transposes of a linear mapping. Orthogonal bases. Gram-Schmidt process.

UNIT IV: Inner product spaces, Adjoint operators. Normal. Unitary and Self-adjoint operators. Spectral theorem for normal operators.

UNIT V: Bilinear Forms. Definition and examples, the matrix of a bilinear form, Orthogonality, Classification of bilinear forms.

DIFFERENTIAL EQUATION

UNIT I: Existences and Uniqueness theorem for the equations of the form $\frac{dy}{dx} = f(x,y)$: Picard`s method of successive approximation. Statements of the existence theorems for a system of first order ordinary differential equations. Independence of solutions of linear differentials equations. Wronskian.

UNIT II: Solution of 2^{nd} order differential equations with variable coefficients. Exact differerential equations. Tranformation of the equation by removal of the first derivative and changing independent variable. Method of variation of paramaters. Simultaneous differential equations and Total differential equations.

UNIT III: Series solution: Solution of Legendre and Bessel's equations. Legendre and Bessel's function of first kind and second kind, their properties.

UNIT IV: Linear partial differential equation of first order: Origin of partial differential equation of 1^{st} order. Standard forms for solution of partial differential equations

by using Lagrange's method. Non-linear partial differential equations: use of standard forms for solution of non linear partial differential equations. Charpit's method. Jacobi's method.

UNIT V: Partial differential equations of second order linear equations with constant coefficients, classification of linear 2^{nd} order equations in two independent variables.

TOPOLOGY

UNIT I: Metric space: Definition and examples of metric space, open and closed sphere, open and closed sets, continuous function, uniform continuity, sequence and their convergence: Cauchy sequence. Complete metric space. Baire category theorem.

UNIT II: Definition and examples of topological space, Closed sets, Closure, Dense sets, neighbourhood; interior, exterior and boundary point; accumulation point and derived sets, continuity and homeomorphism. Bases and sub bases, relative toplogy, Weak topology, Weak * topology, product space.

UNIT III: Basic properties of compactness, Sequentially and countably compact, Tychonoff's theorem, locally compact space. Ascoli's theorem. Lebesgue covering lemma, the Stone-Cech compatification.

UNIT IV: Separation axioms, Lindelof space, T1 – space, Hausdorff space, regular space, normal space, completely regular space, Urysohn's metrization theorem, Tietze extension theorem.

UNIT V: Connected spaces. Connectedness on the real line, totally disconnected, locally connected spaces. Path connectedness.

NUMERICAL COMPUTATIONS

UNIT I: Nonlinear equations, Bisection method, Newton's method and its variants, Fixed-point iteration. Bairstow's method. Convergence analysis.

UNIT II: Definiton and sources of errors. Propagation of errors. Floating-point arithmetic. Backward error analysis. Perturbation analysis.

UNIT III: Systems of linear euations. Gauss elimination, Pivoting. Stability of Gaussian elimination. Cholesky factorization. Ill-conditioning and condition number, Conjugate-gradient algorithm. The eigen value problem. Power method. Rayleigh

quotient. Inverse iteration. Reduction to Hessenberg or tridiagonal form, QR method.

UNIT IV: Approximation of functions. Polynomial interpolation, Hermite interpolation, Interpolation by splines, Least squares curve fitting. Numerical differentiation, Numberical Integration. Extrapolation techniques.

MATHEMATICS PAPER- II

MATHEMTICAL PROGRAMMING

UNIT I: Introduction to Mathematical programming problems formulation, Hyper planes, convex set, convex combination, convex hull, convex polyhedron and simplex, extreme point, Feasible solution, Basic solution, Basic feasible solution and related theorems. Fundamental theorem of linear programming problem, Optimality test, Simple method, two-phase method.

UNIT II: Duality, fundamental theorem of duality, Dual simplex method, comparison of solution of primal and its dual. Revised simplex method, solution of linear programming problem by revised simplex method.

UNIT III: Transportation problems, North-West corner rule, Vogel's approximation method, Optimality test. Assignment problems. Game theory.

UNIT IV: Inventory Control: Deterministic inventory problems with no shortages. Deterministic inventory problems with shortage, EOQ problem with price breaks, Multi-item Deterministic problems.

FUNCTIONAL ANALYSIS

UNIT I: Normed linear spaces, Banach spaces and examples. Quotient space. Continuous functions and bounded linear operators. Finite dimensional normed linear spaces. Equivalent norms. Riesz Lemma.

UNIT II: Fundamental Theorems: Hahn-Banach theorem. Open mapping theorem. Closed graph theorem. Uniform boundedness theorem. Adjoint operator. Weak and Weak* topologies on Banach spaces.

UNIT III: Hilbert space and basic properties. Schwarz inequality. Orthogonal complements. Orthogonal sets. Bessel's inequality. Conjuagate space. Riesz representation theorem.

UNIT IV: Definition and examples of Banach algebra. Complex homomorphisms. Basic properties of Spectra. Gelfand spectral radius formula: Gelfand-Mazur theorem; Group of invertible elements.

UNIT V: Maximal ideals and Homomorphism, semi-simple Banach algebra. Gelfand transform. Involution. C*- algebras, functional calculus in C* - algebras, Gelfand-Nerimark Theorem, Application to non commutative Banach algebras, positive functionals.

TENSOR AND RIGID DYNAMICS

UNIT I: Tensor: Transformation laws of covariant and contravariant tensors. Mixed tensors, rank of tensors, Symmetric and anti-symmetric tensors. Algebraic operations on tensors. Contraction. Inner and outer product of tensors. Quotient law, angle between two vectors.

UNIT II: Christoffel brackets and their transformations. Covariant derivative of co-variant & contravariant tensors. Curl, grad and divergence of tensors with related problems, Laplacian in tensor form.

UNIT III: Generalized coordinates: Lagrange's equation of motion for finite and impulsive forces, conservative forces, small oscillation.

UNIT IV: Hamilton's equations of motion (Canonical form) Principle of virtual work and derivation of Lagrange's and Hamiltonian equation from it. Hamilton's principle, principle of least action.

MATHEMATICAL STATISTICS

UNIT I: Probability: Mathematical and statistical definition, axiomatic approach, sample space, Probability as a set function, Additional and multiplication theorem on probability, conditional probability, repeated trials, Baye`s theorem. Random variable and distribution function.

UNIT II: Mathematical expectation, expectation of sum and product of random variables, variance Tshebysheff lemma, weak law of large numbers, Bernoulli's theorem, moment generating function, characteristic function, central limit theorem.

UNIT III: Probability distribution: Binomial distribution, Negative binomial distribution, Poisson distribution, normal distribution, hyper geometric distribution, Beta, Gamma distribution.

UNIT IV: Regression, Correlation: Correlation – simple, multiple and partial, regression and the theory of least squares, Cauchy – Schwarz`s inequality and limit of correlation coefficient, Multiples regression using matrix.

UNIT V: Sampling distribution and test of significance: Sampling distributions; mean and standard error, level of significance, confidence intervals, test of significance, test Fisher`s Z-transformation, t, x2 and F- distributions.

MATHEMATICAL METHODS

UNIT I: Laplace Transform; Basic properties of Laplcae transform, and its properties. Inverse Laplace transform. Convolution theorem. Application of Laplace transform to solution of ordinary and partial differential equation.

UNIT II: Fourier transform: Fourier integral transform, properties of Fourier transform, Fourier integral, sine and cosine transform, inverse Fourier transform, application to ordinary and partial differential equation.

UNIT III: Variation of a functional, Euler – Lagrange equation, Necessary and sufficient conditions for extremum, Variational methods. Isoperimetric problems and applications.

UNIT IV: Linear integral equation of the first and second kind of Fredholm and Volterra types, Solutions with separable Kernels. Characteristic numbers and Eigen values, Eigen functions, resolvent kernel.

HYDRO DYNAMICS

UNIT I: Kinematics of fluid motion: Path lines, stream lines, rotational motion, irrotational motion. Equation of continuity, equation of motion, boundary conditions, and impulsive motions.

UNIT II: Motion in a Plane: Complex potential, source, sink, doublet, method of images, the theorem of Blausius.

UNIT III: Motion past a sphere, cylinder and their examples.

UNIT IV: Vortex motion: Properties of vortex filament, motion due to a rectilinear vortex and a system of vortices, motion of a vortex line due to the influence of others.

COMMUTATIVE ALGEBRA

UNIT I: Preliminaries on rings and ideals: local and semi local rings; nil radical and Jacobson radical: operations on ideals: extension and contraction ideals. Modules and module homomorphism: sub modules and quotient modules. Operations on sub modules, finitely generated modules, exact sequences.

UNIT II: Tensor product of modules: Existence and Uniqueness of tensor product of two modules; Tensor product of scalars; Exactness properties of the tensor products; Algebras; Tensor product of algebras.

UNIT III: Rings and modules of fractions; Local properties; Extended and contracted ideals in rings of fractions.

UNIT IV: Primary ideals, P-Primary ideals; primary decomposition, minimal primary decomposition, uniqueness theorems, primary sub- modules of a module.

UNIT V: Chain conditions; ascending chain conditions on modules; maximal condition; Noetherian modules; descending chain condition; minimal condition, Artinian modules; their properties; Noetherian rings; Hilbert basis theorem; Artinian rings; structure theorem for Artinian rings.

WAVELETS AND APPLICATION

UNIT I: Fourier Analysis: Fourier transforms in L 1(R). Basic properties of Fourier transforms, Fourier transforms in L 2 (R). Poisson's Summation formula. The Shannon Sampling theorem and Gibbs`s phenomenon. Heisenberg's uncertainty principle.

UNIT II: Continuous wavelet transforms and examples. Basic properties of Wavelet transforms.

UNIT III: The Discrete wavelet transforms. Orthonormal wavelets.

UNIT IV: Multiresolution Analysis: Definition of Multiresolution Analysis and Examples, Properties of scaling functions and Orthonormal wavelet Bases.

UNIT V: Construction of Orthonormal wavelets.

FUZZY SET THEORY AND APPLICATION

UNIT I: Internal arithmetic, multi –level interval numbers. Fuzzy Sets: Basic definitions, α- level sets, Basic operations on Fuzzy sets, types of fuzzy sets, Extension principle for fuzzy sets, t-norms: t- conorms. Fuzzy numbers, Arithmetic with Fuzzy numbers.

UNIT II: Fuzzy relations: Basic properties of fuzzy relations, Fuzzy relations and approximate reasoning. Properties of the Min- Max composition, Fuzzy relation equations. Fuzzy graphs; special fuzzy relations. Fuzzy functions on fuzzy sets, Integration of fuzzy functions. fuzzy differentiation.

UNIT III: Probability theory: Fuzzy measures, Evidence theory; belief measure, plausibility measure and necessity, measures, possibility distribution, possibility

theory; probability of Fuzzy events; possibility theory versus probability theory. Baye's theorem for fuzzy events; probabilistic interpretation of fuzzy sets.

UNIT IV: An overview of classical logic, Maltivalued logics, Linguistic variables, Linguistic modifiers, Truth, Propositions of fuzzy logic, Fuzzy quantifiers, Approximate reasoning. Fuzzy implications, Multi conditional Approximate reasoning. The role of fuzzy relation equations, Interval – valued approximate reasoning.

UNIT V: Fuzzy decision making, multi criteria decision making, Multi stage decision making; Fuzzy ranking methods, Fuzzy controllers, Fuzzy modeling of control parameters. Defuzzification, Washing Machine, Fuzzy Logic control for a predator – Prey system.

OPERATION RESEARCH

UNIT I: Non-Linear propramming: Gereral Non- Linear Programming Problem, Constrained Optimization with Equality Constraints, Contrained Optimization with Inequality Constraints, Kuhn – Tucker Conditions for Gerenal NLPP with m (<n) Constraints, Quadratic programming, Wolfe`s Modified Simplex Method. Beale's Method.

UNIT II: Dynamic programming: The recursive Equation Approach, Characteristics of Dynamics programming. Dynamic Programming Algorithm. Solution of Discrete D.P.P., Solution of L.P.P. by dynamic programming.

UNIT III: Queueing Theory: Queueing system, Elements of Queueing system, Operating Characteristics of Queneing system, Probability Distributions in Queueing system, classification of Queueing Models, Definition of Transient and Steady States, Poisson Queueing system.

UNIT IV: Network Scheduling by PERT/CPM: Net work and Basic Components, Logical Sequencing. Rules of Network Construction, Critical Path Analysis, Probability considerations in PERT, Distinction between PERT and CPM.

UNIT V: Information Theory: A Measure of Information, Entropy – The Expected Information, Entropy as a Measure of Uncertainty, Some properties of Entropy Function, The Communication System, Channel Probabilities, Joint and conditional entropies, Mutual information, Encoding.

FLUID DYNAMICS

UNIT I: Viscosity: Navier stokes equation, vorticity transport equation, circulation, flow between plates through a pipe (circular, elliptic), circular motion between cylinders.

UNIT II: Waves: Progressive waves, long wave and surface wave, stationary wave, Energy of a wave, group velocity.

UNIT III: Boundary layer theory: Boundary layer thickness, Displacement thickness, momentum thickness, Energy thickness, Drag and lift; Boundary layer equations in two dimensional flow, Blausius solution.

UNIT IV: Boundary layer over a wedge, Von Karman`s integral equation, Energy integral equation for two dimensional laminar boundary layers in incompressible flow, Von Karman – pohlhausen method, 'Similar solutions' of the boundary layer equations, separation of the boundary layer flow, plane free jet (two dimensional jet), plane wall jet.

GRAPH THEORY

UNIT I: Graphs, Sub graphs, Walks, paths and circuits, Digraphs, Connected graphs, Paths connected, disconnected graphs and components: Vertex Degrees and graphic sequences. Bipartite graphs. Operations on graphs.

UNIT II: Cut points, bridges and blocks; Block graphs and cut point graphs. Trees, Binary Trees, Spanning Trees, spanning trees in a weighted graph.

UNIT III: Eulerian and Hamiltonian graphs. Line graphs. Factorizations: 1 – Factorizations, 2- Factorization. Covering and critical points.

UNIT IV: Planner graphs. Outer planer graphs. Euler's polyhedron Formula; Kuratowski's theorem. The Chromatic number. Five color theorem, Four color, Conjecture.

UNIT V: Matrix representation of graphs: Adjacency matrix, incidence matrix, Circuit matrix, Fundamental Circuit matrix and rank. Application to a switching Network. Cut set Matrix.

DESIGN AND ANALYSIS OF ALGORITHMS

UNIT I: Models of computation, Algorithm analysis, Order arithmetic, time and spaces complexities, average and worst case analysis, lower bounds.

UNIT II: Algorithm design techniques: divide and conquer, search and traversals, dynamic programming, backtracking, branch and bound.

UNIT III: Sorting and searching algorithms (insertion sort, quick sort, heap sort, merge sort).

UNIT IV: Graph Algorithms: connectivity, shorter path, spanning trees, topological sorting, Algorithms for set union – find problems.

UNIT V: Introduction to NP – completeness, geometric algorithms, approximation algorithms for some NP – complete problems.

FORMAL LANGUAGE AND AUTOMATA THEORY

UNIT I: Alphabets, Languages, Grammars, Finite automata: regular expressions, regular languages.

UNIT II: Context free languages: Pushdown automata. DCFLs, LL (k) and LALR grammars.

UNIT III: Context sensitive languages: Linear bounded automata.

UNIT IV: Turing machine: Recursively enumerable languages.

UNIT V: Operations on formal languages and their properties. Decision questions on languages, Undecidable problems.

DATA BASE MANAGEMENT SYSTEM (DBMS)

UNIT I: Introduction, Comparison between a file – processing system and DBMS, Advantage of a DBMS, data models, schemas and instance, DBMS architecture and data independence, classification of Database Management systems. Data modeling using Entity Relationship model, ER diagrams and design issues, subclass, super class and inheritance, Relational database designs using ER to relational mapping.

UNIT II: Relational model concepts, Relational Algebra operations and solving queries using relational algebra, Solving queries using SQL, views, assertions and triggers.

UNIT III: Design guidelines for relational schemas, Functional dependencies, normal Forms; INF, 2NF, 3NF, BCNF, 4NF, Algorithms for relational database schema design, join dependencies and fifth Normal Forms.

UNIT IV: Concept and event driven programming, Visual basic programming: variables, arrays, collections. Control structures, working with strings, date and times, Developing a simple applications using VB controls.

UNIT V: Basic VB Active X controls and their properties: Text Box control, List Box control, Scroll bar, Slider control, etc., interconnecting VB application with a database.

LARGE SCALE SCIENTIFIC COMPUTATION

UNIT I: Large sparse linear systems, Storage schemes, Review of stationary iterative process, Krylov subspace methods, Conjugate gradients (CG), BICG, MINRES and GMRES.

UNIT II: The Lanczos iteration, From Lanczos to Gauss quadrature, Preconditioning, Error bounds for CG and GMRES.

UNIT III: Effects of finite precision arithmetic, multigrid methods, Multigrid as a preconditioned for Krylov subspace methods.

UNIT IV: Nonlinear systems, Newton`s method and some of its variants, Newton GMRES.

UNIT V: Continuation methods, conjugate direction method, Davidon – Fletcher – Powelll Algorithms.

NETWORK FLOWS AND PARALLEL ALGORITHMS

UNIT I: Graph notations and computer representations, Applications to various disciplines, Worst-case complexity.

UNIT II: Shortest paths, Label setting and label correcting algorithms, Maximum flows, Augmenting path and pre flow push algorithms, Minimum cost flows.

UNIT III: Pseudo polynomial and polynomial time algorithms, Assignments and matching, Bipartite and non-bipartite matching.

UNIT IV: Minimum spanning trees, Convex cost flows and generalized flows, Emphasis on real-life time applications of network flows and state-of-the art algorithms.

UNIT V: Theoretical models of parallel computation: Variants of the PRAM model, Performance of parallel algorithms, Basic Techniques: Balanced trees, recursive doubling, divide and conquer.

COMPUTATIONAL GEOMETRY, MATHEMATICAL MODELING AND NUMERICAL SIMULATION.

UNIT I: Introduction to polygonal and polyhedral geometry. Algorithmic design paradigms and data structures for geometric problems. Deterministic and randomized algorithms. Sorting and searching in geometry.

UNIT II: Specific construction and computation problems including construction of convex hulls, voronoi diagrams and Delaunay triangulation, point location, intersection detection and computation and closest pair computation, Applications to robotic and computer graphics.

UNIT III: Model and its different types, Finite models, Statistical models, Stochastic models, Formulation of a model, Laws and conservation principles, Discrete and continuous models.

UNIT IV: Manipulation into its most respective from, Evaluation of a model, Case studies, Continuum model, Transport phenomena, Diffusion and air pollution models, Microwave heating, Communication and Information technology.

UNIT V: Generation of random numbers and their applications. Design of Simulation experiment and output analysis – Validation of simulator.

ALGEBRAIC TOPOLOGY

UNIT I: Fundamental Group, Homotopy of maps between topological spaces, Homotopy equivalence, Contractable and Simply, Connected spaces, Fundamental Groups of *S1*and *S1XS1*.

UNIT II: Calculation of Fundamental Groups of S", n>1 using Van – Kampen's theorem, Brouwer Fixed Point theorem, Fundamental theorem of Algebra, Vector fields on planar sets, Frobenius theorem for 3x3 matrices.

UNIT III: Covering spaces, Unique Path Lifting theorem, Covering Homotopy theorems, Group of covering transformations, Criterion of lifting maps in terms of Fundamental Groups, Universal covering and its existence, spherical cases of manifolds and topological groups.

UNIT IV: Singular Homology, Reduced Homology, Eilenberg – Steenrod axioms of Homology (no proof for homotopy invariance axiom, excision axiom and exact sequence axiom) and their application, Relation between Fundamental Group and First homology Group.

UNIT V: Calculation of Homology Groups of S", Brouwer Fixed Point theorem for *f:* $\Gamma" \rightarrow \Gamma"$, Application to Spheres, Vector Fields.

STOCHASTIC PROCESSES

UNIT I: Generating Functions, Laplace Transformations, Stochastic Process: Introduction, specification of Stochastic Processes, Recurrent Events, Random walk models: gambler's ruin model. Markov Chain.

UNIT II: Markov Processes in continuous time: introduction, Poisson process, Simple birth process, Simple death process, the simple birth and death process.

UNIT III: The Polya process. Brownian Motion process. Weiner process, Introduction to Epidemic processes: simple epidemics, general epidemics.

UNIT IV: Introduction to Renewal processes, Renewal equation, Renewal theorems, Delayed and Equilibrium renewal process, Introduction to discrete Branching processes: Galton-Watson branching process.

UNIT V: Queueing process: Basic characteristics of queueing system, different performance measures, Steady state solution of Markovian queiueing models: M/M/1, M/M/C. Introduction to diffusion processes: Diffusion limit of random walk, diffusion limit to a discrete branching process.

INDUSTRIAL STATISTICS

UNIT I: Introduction to Statistical Quality Control, Quality of a product, need of quality control, Basic concepts of Process Control. Process Capability and Product Control. Causes of Variation in quality of products. Statistical theory behind control charts, control limits, rational sub groups.

UNIT II: Variables and attributes, charts for variables x and R charts σ charts, CUSUM charts. Charts for attributes: p-chart, np-chart, c-chart, U-charts. Process capability measurements. Process capability ratios-cpk and cp, six-sigma capability.

UNIT III: Problem of lot acceptance, good and bad lots, Producer's risk, consumer's risk, AQL, AOQL, LTPD, ASN, OC function and Curve. Rectifying inspection plan, single sampling plan, Double sampling plan and multiple sampling plan.

UNIT IV: Inventory: Concept of planned inventory policies. Deterministic models – policy when inventory levels are reviewed continuously and demands occur uniformly with and without shortage costs, Economics order quantity, policy for production planning when inventory levels are reviewed periodically. Stochastic models- a single period model with no set cost having zero or non zero initial stock {(s,S) Policy}.

UNIT V: Decision problems in production

Allocation of resource and LPP, transportation problems, critical path methods for production planning like – PERT and CPM. Reliability: Definitions and relationships between survival function, hazard function, hazard rate of a non-negative random variable, parametric distributions – Weibull, gamma, log normal and exponential as life time distributions.

ACTUALRIAL MATHEMATICS

UNIT I: Mortality rates. Probability for age at death. Life tables. Graduation of mortality rates. testing the smoothness of a graduation. Methods of graduation of mortality rates.

UNIT II: Risk models: Individual risk models for a short term, Collective risk models for a single period. Life Insurance: Introduction, Insurance payable at the moment of death, Insurances payable at the moment of death.

UNIT III: Life Annuities: Introduction. Continuous life annuities. Discrete life annuities. Reinsurance: Introduction. Reinsurance arrangements. Distribution and estimation.

UNIT IV: Credibility Theory: Introduction, creditability premium formula, credibility factor, Bayesian credibility. Empirical Bayes credibility theory: Poison/Gamma and Normal/Normal model.

UNIT V: Experience rating: Introduction, the rating of general insurance business, No claims discount (NCD) systems, steady state analysis, the effect of systems on the

propensity to claim Run-off triangles: Introduction, Projection using development factors, adjusting for inflation, the average cost per claim method.

ALGEBRAIC NUMBER THEORY

UNIT I: Divisibility, Primes, Unique factorization of integers, Arithmetical functions, Mobius inversion, congruences. Chinese remainder theorem, primitive roots, Quadratic reciprocity, Binary quadratic forms, Fermat's two square theorem. Lagrange's four square theorem.

UNIT II: Groups and Symmetry: Reviev of groups, subgroups, homomorphisms, cosects, Normal subgroups, products and quotients of groups, Permutation groups.

UNIT III: Group actions and examples, orbits and stabilizers, class equation, Sylow theorems and their application in identifying groups of order 2p and pq. Structure theorem for finitely generated abelian groups.

UNIT IV: Rings and Factorization: Rings, homomorphisms and ideals, quotient rings, integral domains and quotient fields, Unique factorization domains, principal ideal domains, Euclidean domains, algebraic integers in quadratic fields.

UNIT V: Fields, algebraic and transcendental extensions, degree of a field extension, constructions with ruler and compass.

MEASURE THEORY

UNIT I: Algebras and sigma – algebras, measures, measurable space, outer measures, measurable sets. Lebesgue measure and its properties, non-measurable sets. Measurable functions and their properties. Egoroff's theorem.

UNIT II: Lebesgue integration: simple functions, integral of bounded functions over a finite measure, bounded convergence theorem, integral of non negative functions, Fatous's Lemma, monotone convergence theorem, the general Lebesgue integral. Lebesgue convergence theorem.

UNIT III: Signed measures, Hahn and Jordan Decompositions, absolute continuity, Radon Nikodyn theorem, derivatives of signed measured Lp – spaces and their dual.

UNIT IV: Product measures, construction, Fubini's theorem and its applications. Finite and infinite dimensional product spaces. Locally compact spaces, regular measures. Haar measure.

CONTINUUM MECHANICS

UNIT I: Analysis of stress: The continuum concept. Homogeneity. Isotropy, Mass Density, Force and moment of equilibrium. Stress transformation laws, Stress quadric of Cauchy, Principal stresses, stress invariants. Stress ellipsoid.

UNIT II: Deformation and Strain: Continuum Configuration, Deformation and flow concepts, Lagrangian and Eulerian descriptions, Deformation and displacement gradients, Deformation tensors, Finite strain tensors, Small deformation theory, Relative displacement, Linear rotation tensor, Rotation vector, Stretch ratio, Finite strain interpretation. Principal strains, Strains invariants, Cubical Dilatation.

UNIT III: **(a)** Motions and Flow: Material derivative, Velocity, Acceleration, Instantaneous velocity, field, Path lines and stream lines steady motion, Rate of deformation tensors. Velocity tensors and their physical interpretation.

(b) Fundamental laws of Continuum Mechanics: Conservation of mass, Continuity equation, Linear momentum principle, Equation of motion, Equilibrium equations, Moment of momentum principle, Conservation of energy, Energy equation.

UNIT IV: Linear Elasticity: Generalized Hook's law, Strain energy function, Isotropy, Anisotropy, Elastic symmetry, Isotropic media, Elastic constants, Navier-Cauchy equations and Beltrami – Michell equations.

SPACE DYNAMICS

UNIT I: Basic formulas of a spherical triangle – The two-body problem: The motion of the centre of mass. The relative motion. Kepler's equation. Soltuion by Hamilton Jacobi theory. The Determination of Orbits: Laplace's Gauss Methods.

UNIT II: The three Body problem: general three Body problem. Restricted Three Body Problem. Jacobi integral. Curves of Zero velocity. Stationary solutions and their stability. The n- body problem: The motion of the centre of Mass. Classical integrals.

UNIT III: Perturbation: Osculating orbit, Perturbing forces, Secular and periodic perturbations.

UNIT IV: Flgiht Mechanics: rocket performance in Vacuum, vertically ascending paths. Gravity twin trajectories. Multi-stage rockets in a vacuum. Definitions pertinent to single stage rocket. Performance limitations of single stage rockets. Definitions pertinent to multi stage rockets. Analysis of multi-stage rockets neglecting gravity. Analysis of multi-stage rockets including gravity.

UNIT V: Rocket performance with Aerodynamic forces. Short-range non- lifting missiles. Ascent of a sounding rocket, some approximate performance of rocket powered aircraft.