CC/M/EXAM.
2020
MATHEMATICS

PAPER—II

Time : 3 hours] [Full Marks : 250

Note : Question Nos. **1** and **5** are compulsory and out of the remaining, any **three** are to be attempted choosing at least ONE question from each Section. The number of marks carried by a question/part is indicated against it.

SECTION—A

1. Answer *any* **five** of the following questions : 10×5=50

(a) Show that an absolutely convergent series is convergent but the converse is not necessarily true.

(b) Find the singular points of the function of complex variable defined by $f(z) = \frac{1}{\sin\left(\frac{\pi}{z}\right)}$ and also identify its isolated singular points.

(c) Expand $\sinh z$ in a Taylor's series at $z_0 = \pi i$ and show that $\lim_{z \to \pi i} \frac{\sinh z}{z - \pi i} = -1$.

(d) Prove that every subgroup of a cyclic group is cyclic.

(e) Consider the function f defined on $\mathbb{R}^2$ by $f(x, y) = \begin{cases} xy\dfrac{x^2 - y^2}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$ and show that $\frac{\partial^2 f}{\partial x \partial y} \neq \frac{\partial^2 f}{\partial y \partial x}$ at $(0, 0)$.

(f) Obtain all the basic solutions to the system of linear equations :

$$x + 2y + z = 4, \ \ 2x + y + 5z = 5$$

(g) Suppose R is a Euclidean domain and A is an ideal of R. Show that there exists $a \in A$ such that $A = \{ax \mid x \in R\}$.

2. Answer the following questions :

(a) Prove that every group is isomorphic to a permutation group. 20

(b) Suppose G and G' are two groups. Show that a homomorphism $f : G \to G'$ is one-one if and only if $\ker f = \{e\}$, where e is the identity element of G. 15

(c) Let R be the ring of all 3×3 matrices over reals. Show that

$$S = \left\{ \begin{pmatrix} x & x & x \\ x & x & x \\ x & x & x \end{pmatrix} \middle| x \text{ is real} \right\}$$

is a subring of R having a unity different from unity of R. 15

3. Answer the following questions :

(a) Examine uniform convergence of the sequence of functions $\{f_n\}$ defined on $[0, 1]$ by $f_n(x) = n^2 x(1 - x^2)^n$. 20

(b) A real valued function f defined on $[a, b]$ is Riemann integrable over $[a, b]$. Show that $m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$, where m and M are the supremum and infimum of f on $[a, b]$. 15

(c) Show that the function $f(x) = \sin x$, is uniformly continuous on $]0, \infty[$. 15

4. Answer the following questions :

(a) Use simplex method to maximize $z = x + y$, where x and y are non-negative variables subject to the constraints $x + y \geq 3$, $2x + 3y \leq 18$, $x \leq 6$. 20

(b) Let $f(z) = \bar{z} e^{-|z|^2}$, where z is a complex variable. Determine the points at which $f'(z)$ exists and find $f'(z)$ at these points. 15

(c) Let γ denote the boundary of the rectangle whose vertices are $-2-2i$, $2-2i$, $2+i$ and $-2+i$ in the positive direction. Evaluate the contour integral $\int_\gamma \frac{e^{-z}}{z^2+2}\,dz$. 15

SECTION—B

5. Answer *any* **five** of the following questions : 10×5=50

(a) Form the partial differential equation for the family of surfaces $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ with $a+b+c=1$.

(b) Describe the algorithm of numerical solution of ordinary differential equations by Runge–Kutta 4th order method.

(c) Multiply the octal numbers $145{\cdot}23_8$ and $37{\cdot}6_8$ and express the result into decimal.

(d) Solve the partial differential equation :

$$x^2\frac{\partial z}{\partial x}+y^2\frac{\partial z}{\partial y}=(x+y)z$$

(e) State and explain D'Alembert's principle.

(f) Simplify the Boolean expression : $(A+B)(\bar{A}+B)\bar{B}$.

(g) The bob of a simple pendulum executes a circular motion in the horizontal plane. If at the same time, the pendulum is allowed to change its length with time, then find the generalized coordinates required to describe the dynamics.

6. Answer the following questions :

(a) Find the characteristics of the equation $pq=z$ and determine the integral surface which passes through the parabola $x=0,\ y^2=z$. [Here, $p=\frac{\partial z}{\partial x}$ and $q=\frac{\partial z}{\partial y}$] 20

(b) Reduce the following partial differential equation into canonical form and find the general solution :

$$y^2\frac{\partial^2 z}{\partial x^2}-2y\frac{\partial^2 z}{\partial x\partial y}+\frac{\partial^2 z}{\partial y^2}=\frac{\partial z}{\partial x}+6y$$ 15

(c) Solve : $3\frac{\partial^2 z}{\partial x\partial y}-2\frac{\partial^2 z}{\partial y^2}-\frac{\partial z}{\partial y}=\sin(2x+3y)$ 15

7. Answer the following questions :

(a) Find the approximate value of the integral $\int_{0.2}^{1} \sqrt{x}\, dx$ by using Simpson's $1/3$ rule (take step size $h = 0 \cdot 2$). Discuss the merits and demerits of this method. 20

(b) Given, $\log 3 \cdot 141 = 0 \cdot 497068$, $\log 3 \cdot 142 = 0 \cdot 497206$, $\log 3 \cdot 143 = 0 \cdot 497344$, $\log 3 \cdot 144 = 0 \cdot 497483$, $\log 3 \cdot 145 = 0 \cdot 497621$.

Using Newton's forward interpolation formula, find the value of $\log 3 \cdot 14159$. 15

(c) Use bisection method to find the approximate root of $5x^2 + 2x - 3 = 0$, in the interval $[0 \cdot 4, 1]$. (Calculate up to 5 iterations) 15

8. Answer the following questions :

(a) A light inextensible string of length l with two masses m_1 and m_2 attached to the ends passes over a pulley so that the masses are hanging over the surface gravity of the earth. Obtain the expression for the Lagrangian of the system and find the corresponding Lagrange's equations of motion. 20

(b) Deduce the expression for the moment of inertia about the axis passing perpendicularly through the centre of a disc of mass m and radius r. 15

(c) Derive Euler's equation of motion in three dimensions for an inviscid fluid. 15

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