MATHEMATICS

240115

Time : 3 hours

Full Marks: 200

Instruction :

Attempt **all** Sections as directed.

SECTION-A

Answer any ten questions. Each question carries 5 marks.

- **1.** Show that $A \cap B = A (A B)$.
- **2.** If a and b are the roots of the equation $x^2 + px + q = 0$, $p, q \in \mathbb{R}$, then find the equation whose roots are a^2 and b^2 .
- **3.** Let G be a group and let $a, b \in G$. Show that the equation ax = b has an unique solution for $x \in G$.
- **4.** Show that u = (1, 1, 0), v = (1, 3, 2), w = (4, 9, 5) are linearly independent.
- **5.** Let V be the vector space of functions $f : \mathbb{R} \to \mathbb{R}$. Show that $W = \{f(x) : f(1) = 0\}$ is a subspace of V.
- 6. Find the value of λ so that the equation $2x^2 + xy y^2 11x 5y + \lambda = 0$ may represent a pair of straight lines.
- 7. Examine the function $\sin x + \cos x$ for extreme values.
- 8. Determine the value of k for which $y = e^{3x} \cos x$, is a solution of the differential equation y'' 6y' + ky = 0.
- 9. Determine the following :
 - (i) $(00110101)_2 \cdot OR \cdot (01010110)_2 = ?$
 - (*ii*) $(00110101)_2 \cdot \text{XOR} \cdot (01010110)_2 = ?$

22/YY8-2018/MATH

SEAL

10. Forces P, Q, R acting along \overrightarrow{IA} , \overrightarrow{IB} , \overrightarrow{IC} , where I is the incentre (point of intersection of internal angle bisectors) of the triangle ABC are in equilibrium. Show that

$$\frac{P}{\cos\frac{A}{2}} = \frac{Q}{\cos\frac{B}{2}} = \frac{R}{\cos\frac{C}{2}}$$

- 11. A particle P of mass 2 moves the X-axis attracted towards origin O by a force whose magnitude is numerically equal to 8x. Further the particle has a damping force whose magnitude is numerically equal to 8 times the instantaneous speed. If it is initially at rest at x = 20, find (a) the position and (b) the velocity of the particle at any time t.
- 12. Show that the complex function $f(z) = 2x^2 + y + i(y^2 x)$ is not analytic at any point.
- **13.** A card is drawn from a well shuffled pack of cards. Find the probability that it is either a diamond or a king.
- 14. If the first quartile is 142 and semi-interquartile range is 18, find the median (assuming the distribution to be symmetrical).

SECTION-B

Answer any ten questions. Each question carries 7 marks.

- **15.** Given that $z = 2e^{\frac{\pi}{12}i}$ satisfies the equation $z^4 = a(1 + \sqrt{3}i)$, where a is real. Find the value of a. Also find the other three roots of the equation.
- 16. For any two subgroups H and K of a group G, prove the following :
 - (a) $H \cap K$ is a subgroup of G
 - (b) If H is normal in G, then $H \cap K$ is normal in K

22/YY8-2018/MATH

2

17. Determine the rank of the matrix

$$A = \begin{bmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \end{bmatrix}$$

18. Solve by Cramer's rule

 $\begin{cases} x + 2y + 3z = 20\\ 7x + 3y + z = 13\\ x + 6y + 2z = 0 \end{cases}$

19. Find the centre and the radius of the circle

$$x^{2} + y^{2} + z^{2} - 8x + 4y + 8z - 45 = 0, x - 2y + 2z = 3$$

20. Show that the sequence $\{a_n\}$, where

$$a_n = \left[\frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \dots + \frac{1}{(2n)^2}\right]$$

converges to zero.

- **21.** Find the area of the region enclosed by the parabola $y = 2 x^2$ and the line y = -x.
- **22.** Find the solution of the differential equation $x^2y' + 2xy = \cos^2 x$.
- 23. Determine the following :
 - (i) $(110101 \cdot 011)_2 = (?)_8$
 - (*ii*) $(73D5 \cdot 15)_{16} = (?)_{10}$
- 24. A body is resting on a rough inclined plane of inclination α to the horizon, the angle of friction being λ , ($\lambda > \alpha$). If P and Q be the least forces which will respectively drag the body up and down the plane, then prove that

$$\frac{P}{Q} = \frac{\sin(\lambda + \alpha)}{\sin(\lambda - \alpha)}$$

22/YY8–2018/MATH

3

[P.T.O.

25. A particle is projected with a velocity u in a vertical plane so that its range is twice the greatest height attained. Determine its range.

26. Evaluate

$$\oint_C \frac{z}{z^2 + 9} dz$$

where C is the circle |z - 2i| = 4.

- **27.** In a binomial distribution with 6 independent trials, the probability of 3 and 4 successes is found to be 0.2457 and 0.0819 respectively. Find the parameter p and q of the binomial distribution.
- **28.** Given that the mean of a distribution is 5, variance is 9 and the moment coefficient of skewness is -1. Find the first three moments about origin.

SECTION-C

Answer any eight questions. Each question carries 10 marks.

- **29.** Show that a group of order n is cyclic if and only if it has an element of order n.
- **30.** Let u = (1, 1, 3), v = (3, 2, -2), L(u) = (4, 1, 1, 1) and L(v) = (-5, 1, -3, 3). Assume further that L is a linear transformation from \mathbb{R}^3 to \mathbb{R}^4 . If w = (5, 4, 4) and y = (2, 1, -5), find the values of L(w) and L(y).
- **31.** Classify the conic given by $9x^2 + 24xy + 16y^2 2x + 14y + 1 = 0$. Hence determine its axis, vertex, equation of latus rectum and focus.
- 32. Show that

$$\frac{v-u}{1+v^2} < \tan^{-1}v - \tan^{-1}u < \frac{v-u}{1+u^2}, \ 0 < u < v$$

and deduce that

$$\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$$

22/YY8–2018/MATH

- **33.** Find the volume and the area of the curved surface generated by revolving the curve $y = \sin x$, x = 0, $x = \pi$ about the X-axis.
- **34.** Solve the differential equation $(D^2 1)y = x \sin (3x)$.
- **35.** Four equal uniform bars each of weight w and length l are jointed together so as to form a rhombus. This is suspended vertically from one of the joints and a small sphere of weight w' and radius r is kept by balancing it inside the rhombus near the lower vertex. If α is the inclination of upper bars to the vertical, then show using the principle of virtual work that

$$\frac{\sin^3 \alpha}{\cos \alpha} = \frac{w'r}{2(2w+w')l}$$

- **36.** If a planet were suddenly stopped in its orbit supposed circular, show that it would fall into the sun in a time which is $\frac{\sqrt{2}}{8}$ times the period of the planet's revolution.
- 37. Find the CG of the part of the four-cusped hypocycloid

$$\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{a}\right)^{\frac{2}{3}} = 1$$

38. Expand the function

$$f(z) = \frac{1}{(z-1)^2 (z-3)}$$

in Laurent series valid for

(i)
$$0 < |z-3| < 2$$

(ii) $0 < |z-1| < 2$

39. Assume the mean height of soldiers to be 68.22 inches with a variance of 10.8 inches². How many soldiers in a regiment of 1000 would you expect to be (a) over 6 feet tall and (b) below 5.5 feet? Assume heights to be normally distributed.

* * *

22/YY8-2018/MATH

YY8-298

SEAL