MATHEMATICS

240115

Time : 3 hours

Full Marks : 200

***Instruction* :**

*Attempt **all** Sections as directed.*

SECTION—A

Answer *any* **ten** questions. Each question carries 5 marks.

1. Show that $A \cap B = A - (A - B)$.

2. If a and b are the roots of the equation $x^2 + px + q = 0$, $p, q \in \mathbb{R}$, then find the equation whose roots are a^2 and b^2.

3. Let G be a group and let $a, b \in G$. Show that the equation $ax = b$ has an unique solution for $x \in G$.

4. Show that $u = (1, 1, 0)$, $v = (1, 3, 2)$, $w = (4, 9, 5)$ are linearly independent.

5. Let V be the vector space of functions $f : \mathbb{R} \to \mathbb{R}$. Show that $W = \{ f(x) : f(1) = 0 \}$ is a subspace of V.

6. Find the value of λ so that the equation $2x^2 + xy - y^2 - 11x - 5y + \lambda = 0$ may represent a pair of straight lines.

7. Examine the function $\sin x + \cos x$ for extreme values.

8. Determine the value of k for which $y = e^{3x} \cos x$, is a solution of the differential equation $y'' - 6y' + ky = 0$.

9. Determine the following :

(i) $(00110101)_2 \cdot \text{OR} \cdot (01010110)_2 = ?$

(ii) $(00110101)_2 \cdot \text{XOR} \cdot (01010110)_2 = ?$

[redacted]

10. Forces P, Q, R acting along $\overrightarrow{IA}$, $\overrightarrow{IB}$, $\overrightarrow{IC}$, where I is the incentre (point of intersection of internal angle bisectors) of the triangle ABC are in equilibrium. Show that

$$\frac{P}{\cos\frac{A}{2}} = \frac{Q}{\cos\frac{B}{2}} = \frac{R}{\cos\frac{C}{2}}$$

11. A particle P of mass 2 moves the X-axis attracted towards origin O by a force whose magnitude is numerically equal to $8x$. Further the particle has a damping force whose magnitude is numerically equal to 8 times the instantaneous speed. If it is initially at rest at $x = 20$, find *(a)* the position and *(b)* the velocity of the particle at any time t.

12. Show that the complex function $f(z) = 2x^2 + y + i(y^2 - x)$ is not analytic at any point.

13. A card is drawn from a well shuffled pack of cards. Find the probability that it is either a diamond or a king.

14. If the first quartile is 142 and semi-interquartile range is 18, find the median (assuming the distribution to be symmetrical).

SECTION—B

Answer *any* **ten** questions. Each question carries 7 marks.

15. Given that $z = 2e^{\frac{\pi}{12}i}$ satisfies the equation $z^4 = a(1+\sqrt{3}\,i)$, where a is real. Find the value of a. Also find the other three roots of the equation.

16. For any two subgroups H and K of a group G, prove the following :

(a) $H \cap K$ is a subgroup of G

(b) If H is normal in G, then $H \cap K$ is normal in K

17. Determine the rank of the matrix

$$A = \begin{bmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \end{bmatrix}$$

18. Solve by Cramer's rule

$$\begin{cases} x + 2y + 3z = 20 \\ 7x + 3y + z = 13 \\ x + 6y + 2z = 0 \end{cases}$$

19. Find the centre and the radius of the circle

$$x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0,\ x - 2y + 2z = 3$$

20. Show that the sequence $\{a_n\}$, where

$$a_n = \left[\frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \cdots + \frac{1}{(2n)^2}\right]$$

converges to zero.

21. Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.

22. Find the solution of the differential equation $x^2 y' + 2xy = \cos^2 x$.

23. Determine the following :

(i) $(110101 \cdot 011)_2 = (?)_8$

(ii) $(73D5 \cdot 15)_{16} = (?)_{10}$

24. A body is resting on a rough inclined plane of inclination α to the horizon, the angle of friction being $\lambda, (\lambda > \alpha)$. If P and Q be the least forces which will respectively drag the body up and down the plane, then prove that

$$\frac{P}{Q} = \frac{\sin(\lambda + \alpha)}{\sin(\lambda - \alpha)}$$