

ARUNACHAL PRADESH PUBLIC SERVICE COMMISSION, ITANAGAR  
SUBJECT: MATHEMATICS

Time: 3 hours

Full Marks: 200

**Note: Question No. 1 is compulsory and any four from the remaining seven questions. All questions carry equal marks.**

**Q. No. 1.** Attempt any 10 (ten)

10 × 4 = 40

A. If  $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ ,  $B = \{2, 3, 4, 5\}$ ,  $C = \{2, 4, 6, 8\}$  and  $D = \{4, 5, 6, 7\}$ , find

(i)  $B \cup C$ , (ii)  $B \cap D$ , (iii) verify that  $(B \cup C) \cup (A \cap D) = A$ .

B. Let  $Q_+$  be the set of all positive rational numbers and  $*$  a binary operation on  $Q_+$  defined by  $a * b = \frac{ab}{3}$ . Determine the identity element in  $Q_+$  and determine the inverse of  $a$ .

C. If  $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ ,  $(\alpha I + \beta A)^2 = A$  find  $\alpha$  and  $\beta$ .

D. Find  $A^{-1}$  if  $A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$ .

E. Find the volume of the parallelepiped whose edges are represented by  $\vec{a}$ ,  $\vec{b}$ ,  $\vec{c}$  where  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$  are given as  $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ ,  $\vec{b} = 2\hat{i} - \hat{j} - 3\hat{k}$  and  $\vec{c} = 3\hat{i} - \hat{k}$ .

F. Prove that

$$\hat{i} \times (\vec{a} \times \hat{i}) + \hat{j} \times (\vec{a} \times \hat{j}) + \hat{k} \times (\vec{a} \times \hat{k}) = 2\vec{a}.$$

G. Find the differential equation of the family of curves  $y = e^x (A \cos x + B \sin x)$ , where  $A$  and  $B$  are arbitrary constants.

H. Examine the continuity of the following function  $f(x)$  at  $x = 0$ :

$$f(x) = \begin{cases} \frac{xe^x}{1+e^x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0. \end{cases}$$

I. Convert the binary number 1101.1101 to its decimal equivalent.

J. Convert the hexadecimal number 39.B8 to an octal number.

K. Show that the function  $u(x, y) = e^x \cos y$  is harmonic. Determine its harmonic conjugate  $v(x, y)$ .

L. A problem in statistics is given to three students A, B and C whose chances of solving it are  $\frac{1}{2}$ ,  $\frac{3}{4}$  and  $\frac{1}{4}$  respectively. What is the probability that the problem will be solved?

**Q. No.2.** Attempt any 8 (eight)

8 × 5 = 40

A. Expand  $\sin^7 \theta$  in a series of sines of multiples of  $\theta$ .

B. How many generators are there of the cyclic group  $G$  of order 8.

C. Write all the permutations on four symbols 1, 2, 3, 4. Which of these permutations are even?

D. Suppose  $V$  and  $W$  are distinct four dimensional subspaces of a vector space  $V$ , where  $\dim V = 6$ . Find the possible dimensions of  $U \cap W$ .

E. Show that the matrix

$$\begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$$

is Nilpotent and find its index.

F. If  $y = e^{a \sin^{-1} x}$ , Prove that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 + a^2)y_n = 0,$$

where  $y_n$  denotes the  $n$ th derivatives of  $y$ .

G. Solve

$$\frac{d^2 y}{dx^2} - 5 \frac{dy}{dx} + 6y = e^{4x}$$

H. why we do often use flow charts for developing computer programs? Draw a flow chart for finding the square root of a given set of numbers.

I. Find the value of  $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$  where  $C$  is the circle  $|z| = 3$ .

J. If two dice are thrown, what is the probability that the sum is greater than 8?

**Q. No.3.** Attempt any 5 (five)

$$5 \times 8 = 40$$

A. Solve the equation  $2x^3 - x^2 - 18x + 9 = 0$  if two of the roots are equal in magnitude but opposite in sign.

B. Show that the union of two subgroups is a subgroup if and only if one is contained in the other.

C. Find the linear transformation  $T: R^2 \rightarrow R^3$  such that  $T(-1,1) = (-1,0,2)$  and  $T(2,1) = (1,2,1)$ .

D. Find the equation of the plane through the points  $(2, 2, 1)$  and  $(9, 3, 6)$ , and perpendicular to the plane  $2x + 6y + 6z = 9$ .

E. In a S.H.M. the distances of a particle from the middle point of its path at three consecutive seconds are observed to be  $x, y, z$ . Show that the time of a complete oscillation is

$$\frac{2\pi}{\cos^{-1}\left(\frac{x+z}{2y}\right)}$$

F. Four rods of equal weights  $w$  form a rhombus ABCD, with smooth hinges at the joints. The frame is suspended by the point A, and a weight  $W$  is attached to C. A stiffening rod of negligible weight joins the middle points of AB and AD, keeping these inclined at  $\alpha$  to AC. Show that the thrust in this stiffening rod is  $(2W + 4w) \tan \alpha$