ARUNACHAL PRADESH PUBLIC SERVICE COMMISSION, ITANAGAR
SUBJECT: MATHEMATICS

Time: 3 hours Full Marks: 200

Note: Question No. 1 is compulsory and any four from the remaining seven questions. All questions carry equal marks.

Q. No. 1. Attempt any 10 (ten) $10\times 4 = 40$

A. If $A = \{1,2,3,4,5,6,7,8,9\}$, $B = \{2,3,4,5\}$, $C = \{2,4,6,8\}$ and $D = \{4,5,6,7\}$, find
(i) $B\cup C$, (ii) $B\cap D$, (iii) verify that $(B\cup C)\cup(A\cup D) = A$.

B. Let Q_+ be the set of all positive rational numbers and $*$ a binary operation on Q_+ defined by $a*b = \frac{ab}{3}$. Determine the identity element in Q_+ and determine the inverse of a.

C. If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $(\alpha I + \beta A)^2 = A$ find α and β.

D. Find A^{-1} if $A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$.

E. Find the volume of the parallelepiped whose edges are represented by $\vec{a}$, $\vec{b}$, $\vec{c}$ where $\vec{a}$, $\vec{b}$ and $\vec{c}$ are given as $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{c} = 3\hat{i} - \hat{k}$.

F. Prove that

$$\hat{i}\times(\vec{a}\times\hat{i}) + \hat{j}\times(\vec{a}\times\hat{j}) + \hat{k}\times(\vec{a}\times\hat{k}) = 2\vec{a}.$$

G. Find the differential equation of the family of curves $y = e^x(A\cos x + B\sin x)$, where A and B are arbitrary constants.

H. Examine the continuity of the following function $f(x)$ at $x = 0$:

$$f(x) = \begin{cases} \dfrac{xe^{\frac{1}{x}}}{1 + e^{\frac{1}{x}}} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0. \end{cases}$$

I. Convert the binary number 1101.1101 to its decimal equivalent.

J. Convert the hexadecimal number 39.B8 to an octal number.

K. Show that the function $u(x,y) = e^x \cos y$ is harmonic. Determine its harmonic conjugate $v(x,y)$.

L. A problem in statistics is given to three students A, B and C whose chances of solving it are $\frac{1}{2}, \frac{3}{4}$ and $\frac{1}{4}$ respectively. What is the probability that the problem will be solved?

Q. No.2. Attempt any 8 (eight) $8\times 5 = 40$

A. Expand $\sin^7\theta$ in a series of sines of multiples of θ.

B. How many generators are there of the cyclic group G of order 8.

C. Write all the permutations on four symbols 1, 2, 3, 4. Which of these permutations are even?

D. Suppose V and W are distinct four dimensional subspaces of a vector space V, where $\dim V = 6$. Find the possible dimensions of $U \cap W$.

E. Show that the matrix

$$\begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$$

is Nilpotent and find its index.

F. If $y = e^{a\sin^{-1}x}$, Prove that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0,$$

where y_n denotes the nth derivatives of y.

G. Solve

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{4x}$$

H. why we do often use flow charts for developing computer programs? Draw a flow chart for finding the square root of a given set of numbers.

I. Find the value of $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$ where C is the circle $|z| = 3$.

J. If two dice are thrown, what is the probability that the sum is greater than 8?

Q. No.3. Attempt any 5 (five) $5 \times 8 = 40$

A. Solve the equation $2x^3 - x^2 - 18x + 9 = 0$ if two of the roots are equal in magnitude but opposite in sign.

B. Show that the union of two subgroups is a subgroup if and only if one is contained in the other.

C. Find the linear transformation $T: R^2 \to R^3$ such that $T(-1,1) = (-1,0,2)$ and $T(2,1) = (1, 2, 1)$.

D. Find the equation of the plane through the points (2, 2, 1) and (9, 3, 6), and perpendicular to the plane $2x + 6y + 6z = 9$.

E. In a S.H.M. the distances of a particle from the middle point of its path at three consecutive seconds are observed to be x, y, z. Show that the time of a complete oscillation is

$$\frac{2\pi}{\cos^{-1}\left(\frac{x+z}{2y}\right)}$$

F. Four rods of equal weights w form a rhombus ABCD, with smooth hinges at the joints. The frame is suspended by the point A, and a weight W is attached to C. A stiffening rod of negligible weight joins the middle points of AB and AD, keeping these inclined at α to AC. Show that the thrust in this stiffening rod is $(2W + 4w)\tan\alpha$

G. Calculate the mean and standard deviation for the following table giving the age distribution of 542 members.

Age in years:	$20-30$	$30-40$	$40-50$	$50-60$	$60-70$	$70-80$	$80-90$
No. of members:	3	61	132	153	140	51	2

Q. No.4. Attempt any 4 (four) $4 \times 10 = 40$

A. For what values of p and q the set of equations

$$2x + py + 6z = 8$$
$$x + 2y + qz = 5$$
$$x + y + 3z = 4$$

have (i) no solution, (ii) unique solution and (iii) infinite number of solutions?

B. Find the equation of the cylinder whose generators are parallel to

$$\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$$

and whose guiding curve is the ellipse $x^2 + 2y^2 = 1,\ z = 3$.

C. A body, consisting of a cone and a hemisphere on the same base, rests on a rough horizontal table, the hemisphere being in contact with the table; show that the greatest height of the cone so that the equilibrium may be stable, is $\sqrt{3}$ times the radius of the hemisphere.

D. In a bolt factory machines A, B and C manufacture respectively 25%, 35% and 40% of the total. Of their output 5, 4, 2 percents are defective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by machines A, B and C?

E. Show that for any frequency distribution

(i) Kurtosis is greater than unity,

(ii) Coefficient of skewness is less than 1 numerically.

Q. No.5. Attempt any 2 (two) $2 \times 20 = 40$

A. Define moment of a force, couple. Prove that any system of coplanar forces acting on a rigid body is equivalent to a single force acting at an arbitrary chosen point together with a single couple.

B. Define terminal velocity. A particle of mass m is projected vertically under gravity, the resistance of the air being mk times the velocity. Show that the greatest height attained by the particle is

$$\left(V^2/g\right)[\lambda - \log(1+\lambda)],$$

where V is the terminal velocity of the particle and λV is the initial velocity.

C. State the addition and multiplication theorem of expectation. Find

(i) the expectation of the numbers on a die when thrown,

(ii) the expected values of the sum of the numbers of the points on two unbiased dice when thrown.

Q. No.6. 40

State the Mean Value Theorem. Give a geometrical interpretation of this theorem. Using this theorem, prove that

$$\frac{b-a}{1+b^2} \langle (\tan^{-1} b - \tan^{-1} a) \langle \frac{b-a}{1+a^2} \quad \text{if} \quad a \langle b.$$

Also, show that

$$\left(\frac{\pi}{4}+\frac{3}{25}\right) \langle \tan^{-1}\frac{4}{3} \langle \left(\frac{\pi}{4}+\frac{1}{6}\right)$$

Q. No.7. 40

Define (i) p- Discriminant relation,

(ii) c- Discriminant relation,

(iii) Singular solution,

(iv) Tac locus,

(v) Node-locus,

(vi) Cusp-locus.

Find general and singular solution of the differential equation

$$p^3 - 4pxy + 8y^2 = 0,$$

where $p = \frac{dy}{dx}$.

Q. No.8. 40

Give the statement of Laurent's Theorem.

For the function $f(z) = \frac{2z^3+1}{z^2+z}$ find,

(i) a Taylor's series valid in the neighbourhood of the point $z = i$,

(ii)) a Laurent's series valid within the annulus of which centre is the origin.

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