CC/M/EXAM.
2020

PHYSICS

PAPER—II

Time : 3 hours] [Full Marks : 250

Note : Question Nos. **1** and **5** are compulsory and out of the remaining, any **three** are to be attempted choosing at least ONE question from each Section. The number of marks carried by a question/part is indicated against it.

SECTION—A

1. Answer *any* **five** of the following questions : 10×5=50

(a) A 50 MeV beam of protons is fired over a distance of 10 km. If the initial size of the wave packet is $1{\cdot}5\times10^{-6}$ m, what would be the final size upon arrival?

(b) How long will it take for the wave packet of a proton confined to 10^{-15} m to grow to a size equal to the distance between the Earth and the Sun, if this distance is equal to $1{\cdot}5\times10^{8}$ km ?

(c) A particle of mass m, which moves freely in an infinite potential well of length a, has the following wave function at $t = 0$:

$$\psi(x,0) = \frac{A}{\sqrt{a}}\sin\left(\frac{\pi x}{a}\right) + \sqrt{\frac{3}{5a}}\sin\left(\frac{3\pi x}{a}\right) + \sqrt{\frac{1}{5a}}\sin\left(\frac{5\pi x}{a}\right)$$

where A is real constant.

(i) Find A so that $\psi(x,0)$ is normalized.

(ii) If measurements of the energy are carried out, what are the values that will be found?

(d) Write the position operator $\hat{x}$ and momentum operator $\hat{p}$ in terms of ladder operators. Hence find $[\hat{x},\hat{p}]$.

(e) Describe how Stern-Gerlach experiment led to the concept of space quantization. How does it demonstrate existence of magnetic moment and electron spin?

(f) Discuss Zeeman splitting of sodium D lines in weak magnetic field.

(g) What is Raman effect? Explain theoretically, the observed characteristics of the Raman spectrum of a diatomic molecule.

2. Answer the following questions :

(a) A particle of energy E is incident on a one-dimensional potential step defined by

$$V(x) = \begin{cases} 0 & , \text{if } x \le 0 \\ V_0 & , \text{if } x \ge 0 \end{cases}$$

Find the reflection and transmission coefficients when *(i)* $E > V_0$ and *(ii)* $E < V_0$. 20

(b) Normalize the wave function $\langle x \mid \psi \rangle = \begin{cases} Ne^{-ikx} & , x > 0 \\ Ne^{kx} & , x < 0 \end{cases}$. Determine the probability that a measurement of the momentum p finds the momentum between p and $p + dp$ for the wave function. 15

(c) Given $\psi(x) = 5\cos^2\left(\frac{2\pi x}{L}\right) + 2\sin\left(\frac{4\pi x}{L}\right)$. Find the possible values of p and corresponding probabilities for obtaining them. So use the un-normalized function to read off the relative odds. Then rescale them to get the absolute probabilities. 15

3. Answer the following questions :

(a) *(i)* Consider the function $\psi(x) = Ae^{-\frac{m\omega x^2}{2h}}$. Find A to normalize it.

(ii) Consider a harmonic oscillator whose energy in the classical theory is given by $E = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$, so that in the quantum version of the oscillators, the wave function for a state of definite energy obeys

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_E(x)}{dx^2} + \frac{1}{2}m\omega^2 x^2 \psi_E(x) = E\psi_E(x)$$

Show that ψ in *(i)* satisfies this equation with $E = \frac{\hbar\omega}{2}$. 20

(b) Find the energy function ψ_E in a box using $e^{\pm ikx}$ instead of $\sin kx$ and $\cos kx$. 15

(c) Consider a free particle $V(x) = 0$ moving in a ring of circumference L. Let $\psi_n(x)$ be the normalized state of momentum $p = \frac{2\pi n\hbar}{L}$.

(i) Show that $\psi_n(x)$ is also the state of definite energy E and find its energy by showing that it satisfies $H\psi = E\psi$

where (in general) $H\psi(x) \equiv -\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x)$.

(ii) Let $\psi(x,0) = 3\psi_2(x) + 4\psi_3(x)$. Normalize this and find $\psi(x,t)$. 15

4. Answer the following questions :

(a) Distinguish between $\vec{L}$-$\vec{S}$ and $\vec{J}$-$\vec{J}$ coupling schemes in case of two-valence electron system. Under what conditions an L-S and J-J coupling scheme has been observed? Give examples. 20

(b) What is anomalous Zeeman effect? Discuss the Zeeman patterns to the resonance (D_1, D_2) lines of sodium. 15

(c) The absorption spectrum of HCl molecule consists of 4 consecutive lines with wave numbers 83·03 cm^{-1}, 103·73 cm^{-1}, 124·30 cm^{-1} and 145·03 cm^{-1}. What J values do they correspond? Calculate the moment of inertia and the internuclear separation. Also find the region of e.m. spectrum where the spectrum lies. 15

SECTION—B

5. Answer *any* **five** of the following questions : 10×5=50

(a) Cl^{33} decays by positron emission of energy 4·3 MeV. Write the decay equation and find the nuclear radius of the product nucleus.

(b) Calculate the binding energy and specific binding energy for $_{28}Ni^{64} = 63 \cdot 9280\,u$ and $_{29}Cu^{64} = 63 \cdot 9298\,u$, given that $m_p = 1 \cdot 007825\,u$ and $m_n = 1 \cdot 00965\,u$. Compare the stability of Ni^{64} and Cu^{64}.

(c) The linear absorption coefficient μ_e of lead for 1 MeV gamma rays is 74 m^{-1}. Calculate the thickness of lead to reduce the intensity of gamma rays to $\frac{1}{100}$ of its original value and half-value thickness.

(d) Describe the success and failures of the nuclear shell model with suitable examples.

(e) The interplanar spacing of (110)-plane is 2·2 Å for a cubic crystal. Calculate the atomic radius.

(f) Write the characteristics of an ideal OP-AMP. Draw the electronic symbol of an OP-AMP and label its pins.

(g) State and prove De Morgan's theorem. What is meant by De Morganization?

6. Answer the following questions :

(a) A simple cubic crystal illuminated with X-rays of wavelength 0·09 nm is rotated and the first-order Bragg reflection occurs at a minimum glancing angle of 8·8°. Which set of crystal planes is responsible for this reflection? Find the spacing between these planes and the angle for the first-order reflection from the (110) crystal planes. 20

(b) Describe band theory of solids on the basis of Kronig-Penney model. Distinguish among conductors, semi-conductors and insulators. 15

(c) Discuss the Fermi level in *n*-type and *p*-type semiconductors with temperature. 15

7. Answer the following questions :

(a) What is a JFET? Describe the principle of operation of an FET. 20

(b) Describe in short, high temperature T_C superconductivity. 15

(c) The critical fields for a sample are $1{\cdot}4\times10^5$ Am^{-1} and $4{\cdot}2\times10^5$ Am^{-1} at $T_C = 14$ K and 13 K respectively. What are the transistor temperatures and critical field at 0 K and 4·2 K? 15

8. Answer the following questions :

(a) In a star there is the following carbon–nitrogen cycle to produce energy :

$p + {}_6C^{12} \rightarrow {}_7N^{13} \rightarrow {}_6C^{13} + e^+ + \nu$

$p + {}_6C^{13} \rightarrow {}_7N^{14}$, $p + {}_7N^{14} \rightarrow {}_7O^{15} \rightarrow {}_7N^{15} + e^+ + \nu$

$p + {}_7N^{15} \rightarrow {}_6C^{12} + {}_2He^4$

Calculate the total energy released in the above cycle by taking

$m_H = 1{\cdot}007825$ u, $m_{He} = 4{\cdot}0026034$ u, $m_e = 5{\cdot}4\times10^4$ u and 1u = 931·5 MeV. 20

(b) Explain the difference between nuclear fission and nuclear fusion. Outline the properties of nuclear fission. 15

(c) Calculate the threshold energy for the following photon–disintegration reaction—

$$ {}_1H^2 + \gamma(h\nu) \rightarrow {}_1H^1 + n \rightarrow p + n $$

Given that $m(d) = 2{\cdot}0140$ u, u = 940 MeV 15

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