ARUNACHAL PRADESH PUBLIC SERVICE COMMISSION

SUBJECT: - MATHEMATICS

Time: 3 (Three) hours **Full marks: 100**

Note:

1. **Question No.1 is compulsory.**
2. **Attempt five questions from the rest**

Q.No. 1. Write *the correct answer (Attempt any five only)* **5x2 =10**

(i) The function $f(x) = |x-1|$ is

(a) continuous at $x = 1$
(b) not derivable at $x = 1$
(c) both (a) and (b) are true
(d) none of these

(ii) The value of $\lim_{x\to 0}\left(\frac{1-\cos x}{x^2}\right)$ is

(a) $\frac{1}{2}$
(b) $\frac{1}{3}$
(c) $\frac{1}{4}$
(d) 1

(iii) The eigen values of the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ are

(a) 1, 6
(b) 2, 3
(c) 3, 4
(d) 4, 6

(iv) If $L\{F(t)\} = f(p)$, then

(a) $L\{F(at)\} = \frac{1}{a} f\left(\frac{p}{a}\right)$
(b) $L\{F(at)\} = \frac{1}{a} f(p)$
(c) $L\{F(at)\} = f\left(\frac{p}{a}\right)$
(d) $L\{F(at)\} = \frac{1}{a^2} f\left(\frac{p}{a}\right)$

(v) Consider the mapping $T: R^2 \to R^2$ such that $T(1,0) = (1,1)$ and $T(0,1) = (-1,2)$. Then

(a) $T(x,y) = (x+y, x-2y)$
(b) $T(x,y) = (x+y, x+2y)$
(c) $T(x,y) = (x-y, x-2y)$
(d) $T(x,y) = (x-y, x+2y)$

(vi) The function $f(z) = xy + iy$ is

(a) not continuous
(b) analytic
(c) continuous but not analytic
(d) none of these

(vii) The value of the integral $\frac{1}{2\pi i}\int_{|z|=3} \frac{e^{zt}}{z^2+1} dz, t > 0$ is

(a) $\cos t$
(b) $\cos 2t$
(c) $\sin t$
(d) $\sin 2t$

(viii) If $P(A)=\frac{1}{3}, P(B)=\frac{3}{4}$ and $P(A\cup B)=\frac{11}{12}$, then $P(A\cap B)=$

(a) $\frac{2}{9}$ (b) $\frac{1}{2}$

(c) $\frac{1}{6}$ (d) $\frac{2}{3}$

Q. No.2. Attempt any 3 (Three) parts: **6×3=18**

(i) Find the bilinear transformation which transform the points $z = 2, i, -2$ into the points $w = 1, i, -1$.

(ii) Let Q^+ be the set of positive rational numbers. Define an operation * on Q^+ as $a*b=\frac{ab}{3}$ for $a,b\in Q^+$. Verify that $(Q^+,*)$ is an abelian group.

(iii) Find the angle through which the axes are rotated in order that the expression $ax^2+2hxy+by^2$ is transformed into an expression not containing the term xy.

(iv) Evaluate: $\frac{\Delta^2 x^3}{E^2 x^3}$.

(v) Show that the maximum value of the function (x-2)(x-3)(x-4) is $\frac{2}{3\sqrt{3}}$ at $x=3-\frac{1}{\sqrt{3}}$.

Q. No.3. Attempt any 3 (Three) parts: **6×3=18**

(i) Show that the function $e^x(\cos y+i\sin y)$ is analytic.

(ii) Find the binary equivalent of $(23)_{10}$.

(iii) Convert $(D6C1)_{16}$ into decimal number.

(iv) Solve: $\frac{xdx}{y^2z}=\frac{dy}{xz}=\frac{dz}{y^2}$.

(v) If $\sigma=\begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 1 & 4 & 3\end{pmatrix}$, then find σ^3.

Q. No.4. Attempt any 2 (Two) parts: **9×2=18**

(i) Find the complete integral of $2xz-px^2-2qxy+pq=0$ (using Charpit's method).

(ii) Let $f(x,y)=\begin{cases}\frac{xy(x^2-y^2)}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$

Show that $f_{xy}\neq f_{yx}$ at (0, 0). Also find f_{xx}, f_{yy} at (1, 1).

(iii) Given $\frac{dy}{dx}=y-x$ where y(0) = 2, find y(0.1) and y(0.2) correct to four decimal places by RungeKutta method.

Q. No.5. Attempt any 2 (Two) parts: **9×2=18**

(i) Evaluate: $\int_0^6 \frac{dx}{1-x^2}$ by using Simpson's $\frac{1}{3}$ rule.

(ii) State and prove D'Alembert's Principle. Also, deduce the general equation of motion of rigid body from D'Alembert's Principle.

(iii) Derive euler's dynamical equations of motion for inviscid fluid.

Q. No.6. Attempt any 2 (Two) parts: **9×2=18**

(i) State Lagrange's interpolation formula. Apply it to find $f(5)$ given that $f(1)=2, f(2)=4, f(3)=8, f(4)=16, f(7)=128$ and explain why the result differs from 2^5.

(ii) Solve the following minimal assignment problem

			Man			
		I	II	III	IV	V
	A	1	3	2	3	6
	B	2	4	3	1	5
Job	C	5	6	3	4	6
	D	3	1	4	2	2
	E	1	5	6	5	4

(iii) Prove that every finite integral domain is a field.

Q. No.7. Attempt any 2 (Two) parts: **9×2=18**

(i) If $u = x^2 \tan^{-1}\frac{y}{x} - y^2 \tan^{-1}\frac{x}{y}, xy \neq 0$, prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{x^2 - y^2}{x^2 + y^2}$.

(ii) If $\vec{a} = (2,1,-1), \vec{b} = (1,-1,0), \vec{c} = (5,-1,1)$, find the unit vector parallel to $\vec{a} + \vec{b} - \vec{c}$, but in the opposite direction.

(iii) Reduce the matrix $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 2 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ to echelon form.

Q. No.8. Attempt any 2 (Two) parts: **9×2=18**

(i) Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$.

(ii) Show that the particle executing simple harmonic motion requires $\frac{1}{6}$ th of its period to move from the position of maximum displacement to one in which the displacement is half the amplitude.

(iii) A heavy uniform rod of length 'a' rests with one end against a smooth vertical wall, the other end being tied to a point of the wall by a string of length 'l'. Prove that the rod may remain in equilibrium at an angle θ to the wall, given by

$$\cos^2\theta = \frac{l^2 - a^2}{3a^2}$$

Q. No.9 . Attempt any 1 (one) parts: **18**

(i) Prove that (i) $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx = \beta(m,n)$

(ii) $\Gamma(m)\Gamma(1-m) = \frac{\pi}{\sin m\pi}, 0 < m < 1.$

(ii) Applying Stoke's theorem prove that $\int_C ydx + zdy + xdz = -2\sqrt{2\pi}\, a^2$, where C is the curve given by $x^2+y^2+z^2-2ax-2ay=0, x+y=2a$ and begins at the point $(2a, 0, 0)$ and goes at first below the xy-plane.

Q. No.10. Attempt any 2 (Two) parts: **9×2=18**

(i) Reduce the equation $y''+Py'+Qy=R$, where P, Q, R are the functions of x, to the form $\frac{d^2v}{dx^2}+Iv=S$, where $I = Q-\frac{1}{4}P^2-\frac{1}{2}\frac{dP}{dx}, S=\frac{R}{u}$.

(ii) Find the shortest distance from the origin to the hyperbola $x^2+8xy^2+7y^2=225, z=0$ (using Lagrange's methods of multipliers).

(iii) If the velocity of an incompressible fluid at the point (x,y,z) is given by $\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{3z^2-r^2}{r^5}\right)$

Prove that the liquid motion is possible and that the velocity potential is $(\cos\theta/r^2)$. Also determine the streamlines.

Q. No.11. Attempt any 2 (Two) parts: **9×2=18**

(i) A particle is projected along the inner side of a smooth vertical circle of radius a, the velocity at the lowest point being u. Show that if $2ga < u^2 < 5ga$, the height point and will describe a parabola whose latus rectum is $\frac{2(u^2-2ga)^2}{27a^2g^3}$.

(ii) A uniform ladder is in equilibrium with one end resting on the ground and the other against a vertical wall, if the ground and wall be both rough, the coefficient of friction being μ and μ', respectively, and if the ladder be on the point of slipping at both ends, show that the inclination of the ladder to the horizon is given by $\tan\theta = \frac{1-\mu\mu'}{2\mu}$.

(iii) Show that if A^{ij} is a symmetric tensor, then

$$A^j_{i,j} = \frac{1}{\sqrt{g}}\frac{\partial}{\partial x^j}\left(A^j_i\sqrt{g}\right) - \frac{1}{2}A^{jk}\frac{\partial g_{jk}}{\partial x^i}.$$

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