# ARUNACHAL PRADESH PUBLIC SERVICE COMMISSION <br> Subject: Statistics 

Time Allowed: 3(three) Hours
Maximum Marks: 100
Note: Question 1 is compulsory and attempts any two questions from each section.
Q.1. (a) Fill in the blanks
(i) If 5 is subtracted from each observation of a set, the mean of the observation is reduced by $\qquad$
(ii) If in a series, 20 percent values are less than 35 , then $=35$.
(iii) In most of the situations, index numbers are the. $\qquad$ averages.
(iv) If $\rho=0$, the two regression lines are at the angel of. $\qquad$
(v) For a leptokurtic curve, the relation between $\mu_{4}$ and $\mu_{2}$ is $\qquad$
(b) $\quad \mathrm{X}$ and Y are two random variables with variance $\sigma^{2}{ }_{X}$ and $\sigma^{2}{ }_{Y}$ respectively and r is the coefficient of correlation between them. If $\mathrm{U}=\mathrm{X}+\mathrm{kY}$ and $\mathrm{V}=\mathrm{X}+\left(\sigma_{\mathrm{X}} / \sigma_{\mathrm{Y}}\right) \mathrm{Y}$, find the value of k so that U and V are uncorrelated.
(c) By using the following data, find out the two lines of regression.

$$
\begin{equation*}
\Sigma \mathrm{X}=250, \Sigma \mathrm{Y}=300, \Sigma \mathrm{XY}=7900, \Sigma \mathrm{X}^{2}=6500, \Sigma \mathrm{Y}^{2}=10000, \mathrm{~N}=10 \tag{5}
\end{equation*}
$$

(d) A personal manager of a large chain of retail stores took a sample of 5 sales personal at random and recorded data regarding their experience in years and sales in Rs. as follows.

| Experience (Years) X: | 2 | 4 | 6 | 8 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Sales Performance (Rs. Lac) Y: | 20 | 12 | 18 | 10 | 40 |

Fit the data in form of the line $\mathrm{Y}=\mathrm{a}+\mathrm{bX}$.

## SECTION A

Q. 2 (a) The following numbers give the weights of 55 students of each class. Prepare a suitable frequency table:

| 42 | 74 | 40 | 60 | 82 | 115 | 41 | 61 | 75 | 83 | 63 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 53 | 110 | 76 | 84 | 50 | 67 | 65 | 78 | 77 | 56 | 95 |
| 68 | 69 | 104 | 80 | 79 | 79 | 54 | 73 | 59 | 81 | 100 |
| 66 | 49 | 77 | 90 | 84 | 76 | 42 | 64 | 69 | 70 | 80 |
| 72 | 50 | 79 | 52 | 103 | 96 | 51 | 86 | 78 | 94 | 71 |

Draw the histogram and ogive chart from the above data.
(b) Show that in a discrete series if deviations are small to compare with mean M so that $(x / M)^{3}$ and higher power of $(x / M)$ are neglected, we have
(i) $\mathrm{G}=\mathrm{M}\left(1-\frac{1}{2} \frac{\sigma 2}{M^{2}}\right)$
(ii) $\quad \mathrm{H}=\mathrm{M}\left(1-\frac{\sigma 2}{M^{2}}\right)$

Where M is the arithmetic mean, H the geometric mean and $\sigma$ is the standard deviation of the distribution.
Q. 3 (a) For the following continuous frequency distribution, calculate arithmetic mean (AM), geometric mean (GM) \& harmonic mean (HM) and verify that $\mathrm{AM} \geq \mathrm{GM} \geq \mathrm{HM}$.

| Class Interval | $29-31$ | $31-33$ | $33-35$ | $35-37$ | $37-39$ | $39-41$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 14 | 7 | 19 | 11 | 13 | 6 |

(b) For a distribution of 250 heights, calculations showed that the mean, standard deviation, $\beta_{1}$ and $\beta_{2}$ were 54 inches, 3 inches, 0 and 3 inches respectively. It was however, discovered on checking that the two items 64 and 50 in the original data were wrongly written in place of the correct values 62 and 52 inches respectively. Calculate the correct frequency constants.
Q. 4 (a) Lives of two models of refrigerators turned in for new models in a recent survey are given in the adjoining table. What is the average life of each model of these refrigerators and which model shows more uniformity?

| Life <br> (No. of Years) | Model A | Model B |
| :---: | :--- | ---: |
| $0-2$ | 5 | 2 |
| $2-4$ | 16 | 7 |
| $4-6$ | 13 | 12 |
| $6-8$ | 7 | 19 |
| $8-10$ | 5 | 9 |
| $10-12$ | 4 | 1 |

(b) What is meant by a time series? Describe the different methods for determining trend in a time series. Examine critically merits and demerits of these methods.

## SECTION B

Q. 5 (a) (i) if $\mathrm{Z}=\mathrm{aX}+\mathrm{bY}$ and r is the correlation coefficient between X and Y , show that $\sigma_{Z}^{2}=a^{2} \sigma_{X}^{2}+b^{2} \sigma_{Y}^{2}+2 a b r \sigma_{X} \sigma_{Y}$
(ii) Show that the correlation coefficient $r$ between two random variables $X$ and $Y$ is given by

$$
\begin{equation*}
\mathrm{r}=\left(\sigma_{X}^{2}+\sigma_{Y}^{2}-\sigma_{X-Y}^{2}\right) / 2 \sigma_{X} \sigma_{Y} \tag{5,5}
\end{equation*}
$$

(b) What is meant by Cost of Living Index Number in India? Explain how you would construct the Cost of Living Index. What are the criticisms leveled against this index?
Q.6. (a) Compute seasonal indices by link relative method for the following data.

|  | Years |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Quarter | 1996 | 1997 | 1998 | 1999 | 2000 |
| I | 30 | 35 | 31 | 31 | 34 |
| II | 26 | 28 | 29 | 31 | 36 |
| III | 22 | 22 | 28 | 25 | 26 |
| IV | 31 | 36 | 32 | 35 | 33 |

(b) Define central moments of a frequency distribution. Obtain the relation between the central moments $\left(\mu_{r}\right)$ of order $r$ in terms of moments $\mu_{r}$ about any point A. Hence obtain the relations for $\mu_{2}, \mu_{3}$ and $\mu_{4}$. Also discuss about the Sheppard's correction. What will be the corrections for the first four moments?
(10)
Q. 7 (a) Calculate Fisher's Ideal Index from the following data and prove that it satisfy both Time Reversal and Factor Reversal test.

|  | Base Year |  | Current Year |  |
| :---: | :---: | :---: | :---: | :---: |
| Items | Price | Quantity | Price | Quantity |
| A | 6 | 50 | 10 | 60 |
| B | 2 | 100 | 2 | 120 |
| C | 4 | 60 | 6 | 60 |

(b) Show that for any frequency distribution:
(i) Bowley's coefficient of skewness is less than unity.
(ii) Karl Pearsons's coefficient of skewness lies between -3 and 3 .

