CC/M/EXAM.
2020
STATISTICS

PAPER—II

Time : 3 hours] [Full Marks : 250

Note : Question Nos. **1** and **5** are compulsory and out of the remaining, any **three** are to be attempted choosing at least ONE question from each Section. The number of marks carried by a question/part is indicated against it.

SECTION—A

1. Answer *any* **five** of the following questions : 10×5=50

(a) A supermarket has two girls serving at the counters. The customers arrive in a Poisson fashion at the rate of 12 per hour. The service time for each customer is exponential with a mean of 6 minutes. Find *(i)* the probability that an arriving customer has to wait for service, *(ii)* the average number of customers in the system and *(iii)* the average time spent by a customer in the supermarket.

(b) A machine is set to deliver packets of given weight. Ten samples of size 5 were examined and the following results were obtained :

Sample No.	**Mean**	**Range**
1	43	5
2	49	6
3	37	5
4	44	7
5	45	7
6	37	4
7	51	8
8	46	6
9	43	4
10	47	6

Calculate the values for the central line and control limits for the mean chart and the range chart. Comment on the state of control.

(c) Prove that both Fisher's ideal index number and Marshall–Edgeworth index number lie between Laspeyres and Paasche's index numbers.

(d) Calculate the general fertility rate, total fertility rate and the gross reproduction rate from the following data, assuming that for every 100 girls, 160 boys are born :

Age of women	15–19	20–24	25–29	30–34	35–39	40–44	45–49
No. of women	218619	198732	161800	145362	128109	106211	86753
Age-SFR (per 1000)	98	169·6	158·2	139·7	98·6	42·8	16·9

(e) Describe the problem of multicollinearity in regression modeling and discuss how it can be detected. What are its consequences?

(f) Write the control charts for standard deviation and control charts for number of defects when (i) standards are given and (ii) standards are not given.

(g) Find an optimum solution to the following transportation problem, for transporting the goods at minimum cost :

Factory ↓	Warehouse				
	D	E	F	G	Capacity
A	42	48	38	37	160
B	40	49	52	51	150
C	39	38	40	43	190
Demand	80	90	110	160	

2. Answer the following questions :

(a) A manufacturer is offered two machines A and B. Machine A is priced at ₹5,000 and running costs are estimated at ₹800 for each of the first five years, increasing by ₹200 per year in the sixth and subsequent years. Machine B, which has the same capacity as A, costs ₹2,500 but will have running costs of ₹1,200 per year for six years, increasing by ₹200 per year thereafter.

If money is worth 10% per year, which machine should be purchased? (Assume that the machine will eventually be sold for scrap at a negligible price.) 20

(b) Elaborate briefly the terms 'censoring' and 'truncation'. Assuming the underlying survival time to be from an exponential distribution with parameter λ, derive the likelihood functions when (i) the lifetime data is right censored and (ii) the lifetime data is left truncated. 15

(c) Explain what you understand by 'seasonal variation' in a time series.

A company manufactures bicycles. You are supplied with monthly production figures of the company for last 10 years. Explain clearly the procedure to be adopted to construct 'seasonal indices' by the 'ratio to moving average' method. 15

3. Answer the following questions :

(a) Describe the various components of a life table. How is the expectation of life at birth determined from a life table? How can it be calculated from Census data? 20

(b) Throw some light on the concept of failure rate and reliability function of a r.v. T and hence, obtain the failure rate and reliability function of the exponential and Weibull failure models. Also, state the conditions under which failure rate of Weibull failure model is increasing, decreasing and constant. 15

(c) Consider the t.p.m. of the Markov chain $\{X_n, n \geq 1\}$ with state space $\{0, 1, 2, 3, 4\}$ as

$$\begin{pmatrix} 0\cdot4 & 0\cdot6 & 0 & 0 & 0 \\ 0\cdot5 & 0\cdot5 & 0 & 0 & 0 \\ 0 & 0 & 0\cdot3 & 0\cdot7 & 0 \\ 0 & 0 & 0\cdot5 & 0\cdot4 & 0\cdot1 \\ 0 & 0 & 0 & 0\cdot8 & 0\cdot2 \end{pmatrix}$$

Classify the states of the Markov chain and determine if the states are ergodic. 15

4. Answer the following questions :

(a) Five salesmen are to be assigned to five districts. Estimates of sales revenue (in thousands) for each salesman is given as follows : 20

Salesman	**District**				
	I	**II**	**III**	**IV**	**V**
A	32	38	40	28	40
B	40	24	28	21	36
C	41	27	33	30	37
D	22	38	41	36	36
E	29	33	40	35	39

Find the assignment pattern that maximizes the sales revenue.

(b) Prepare price and quantity index numbers from the following data, by using *(i)* Laspeyres method, *(ii)* Paasche's method and *(iii)* Fisher's method : 15

Commodity	**Base Year**		**Current Year**	
	Price (in ₹)	**Quantity (in kg)**	**Price (in ₹)**	**Quantity (in kg)**
A	25	49	2,000	50
B	22	18	1,200	30
C	54	16	1,320	44
D	20	40	1,350	45
E	18	30	630	15

(c) Describe the experimental methods usually used for estimating the reliability of a test and indicate their relative merits. 15

SECTION—B

5. Answer *any* **five** of the following questions : 10×5=50

(a) A test of 36 items has a validity coefficient of 0·45 with respect to a given criterion and a reliability coefficient of 0·75. The test is extended to 108 items. Obtain the validity coefficient of the extended test.

(b) The population of a state at ten-yearly intervals is given below :

Year	1881	1891	1911	1921	1931	1941	1951	1971
Population (in million)	3·9	5·3	7·3	9·6	12·9	17·1	23·2	30·9

Fit a curve of the form $y = ab^x$ and estimate the population for 1961.

(c) Distinguish between specification limits and natural tolerance limits.

(d) For the game with payoff matrix

$$\text{Player } B \overset{\text{Player } A}{\begin{bmatrix} -1 & 2 & -2 \\ 6 & 4 & -6 \end{bmatrix}}$$

Determine the best strategies for players *A* and *B* and also the values of the game for them. Is this game fair and strictly determinable?

(e) State and prove decomposition property of a Poisson process.

(f) Consider the two-equation system :

$$\beta_{11}y_{1t} + \beta_{12}y_{2t} + \gamma_{11}x_{1t} + \gamma_{12}x_{2t} = u_{1t}$$

$$\beta_{21}y_{1t} + \beta_{22}y_{2t} + \gamma_{21}x_{1t} + \gamma_{22}x_{2t} = u_{2t}$$

Determine the identification states of each equation under the restrictions $\gamma_{11} = 0$, $\gamma_{21} = 0$.

(g) The age-specific death rates of Poland and Sweden in 1957 and the age distribution of a standard population are given below :

Age (in years)	**Death rate per thousand**		**Standard population (in thousands)**
	Poland	**Sweden**	
0–4	18·870	4·348	119·9
5–14	0·759	0·465	206·9
15–24	1·385	0·767	183·2
25–34	2·098	1·075	147·9
35–44	3·326	1·882	120·5
45–54	7·006	4·669	93·9
55–64	18·111	12·477	70·8
65–74	45·795	34·060	40·5
75 and over	124·258	116·433	16·4

Compute the standardized death rates of the two countries and then compare their mortality situation.

6. Answer the following questions :

(a) Describe the double sampling plan for attributes in detail and derive the expressions for O.C. curve, consumer's and producer's risk, ASN and ATI. Also, discuss a few points of difference between single and double inspection plan. 20

(b) Discuss Cost of Living Index Number. What are the uses and limitations of a Cost of Living Index Number? Describe in detail how it is constructed in general. 15

(c) An automobile company manufactures around 150 scooters. The daily production varies from 146 to 154 depending upon the availability of raw materials and other working conditions :

Production (per day)	146	147	148	149	150	151	152	153	154
Probability	0·04	0·09	0·12	0·14	0·11	0·10	0·20	0·12	0·08

The finished scooters are transported in a specially arranged lorry accommodating 150 scooters using the following random numbers :

80 81 76 64 43 18 26 10 12 65 68 69 61 57

Simulate the process and find out the average number of scooters waiting in the factory. 15

7. Answer the following questions :

(a) Name and then discuss the detailed functions and activities of any two central statistical organizations in India. 20

(b) Solve the following LPP : 15

Minimize : $Z = 6x_1 + 3x_2$

subject to $x_1 + x_2 \geq 1$

$2x_1 - x_2 \leq 1$

$3x_2 \leq 2$

$x_1, x_2 \geq 0$

(c) Discuss the fundamental problem of Economic Order Quantity (EOQ) (assuming lead time to be zero) and derive the formulae for optimum lot size, optimum number of orders placed per unit time, economic review period and total annual expected cost. 15

8. Answer the following questions :

(a) Discuss the Ordinary Least Squares (OLS) and Generalized Least Squares (GLS) method of estimation and discuss a few points of difference between them. Also, show that OLS estimators are BLUEs. 20

(b) Elucidate AR(p) series in detail. For an autoregressive series of order 2, say

$$y_{t+2} + ay_{t+1} + by_t = \varepsilon_{t+2}, \; |b| < 1$$

Show that the correlogram of order k is given by

$$\rho_k = \frac{p^k \sin(k\phi + \psi)}{\sin \psi}$$

where $p = +\sqrt{b}$, $\tan\psi = \dfrac{1+p^2}{1-p^2}\tan\theta$, $\cos\theta = \dfrac{-a}{2p}$. 15

(c) Explain the method of population projection using logistic curve and state its limitations. 15

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