ARUNACHAL PRADESH PUBLIC SERVICE COMMISSION

STATISTICS: PAPER - II

Time- 3 (Three) Hours | Full Marks- 100

Question No 1 is compulsory and answer any four from the rest.

1) Briefly describe or define any 10 (ten) of the following. $2 \times 10 = 20.$

(i) Conditions/ relationship between parameters of a BIBD.

(ii) Difference between complete and partial confounding.

(iii) Uniformly most powerful unbiased critical region

(iv) Minimax rule

(v) Human development index

(vi) Two stage sampling

(vii) Kaplan Meier estimator

(viii) Smoothing in Time Series analysis

(ix) Markov chain

(x) Chapman Kolmogorov equation.

(xi) Transportation problem

(xii) Empirical distribution function in non parametric analysis

(xiii) Natural conjugate family of prior distribution

(xiv) Difference between GRR and NRR in the study of population.

(xv) Concept of pointers in C++ programming.

2) Answer any 4 (four) of the following. $5 \times 4 = 20$

(i) What is posterior distribution? Is it an improvement over the prior distribution? If so, explain how. Also explain the concept of loss function and Bayes risk.

(ii) Let $X_1, X_2, \ldots, X_n$ be a random sample from Poi (λ). Let the prior p.d.f. of λ be $g(\lambda) = e^{-\lambda}$, $\lambda > 0$ and the loss function be squared error. Find the Bayes estimate of $e^{-\lambda}$.

(iii) Why are non parametric procedures useful? Discuss a non parametric test for testing randomness.

(iv) Prove that if a minimax estimator of a parametric function is unique, then it must be admissible.

(v) Write a note on Mahalanobis D^2 statistic and its applications.

(vi) Discuss a method for construction of abridged life table.

3) Answer any 4 (four) of the following. $5 \times 4 = 20$

(i) Distinguish between objects and classes in object oriented programming.

(ii) Discuss various approaches in estimation of national income.

(iii) Arrivals to a counter occur according to Poisson process with λ=2 per hour. What is the probability that no customer arrives between 8AM and 10AM? What is the probability that two or more customers arrive between 12 noon and 1PM?

(iv) Obtain an expression for variance of the Hurvitz Thompson estimator.

(v) Consider a k out of n system, each of whose components has an exponential life time distribution with mean $(1/\lambda)$. Find the variance of the time of the system.

(vi) Distinguish between stable and stationary population.

4) Answer any 2 (two) of the following. $10 \times 2 = 20$

(i) What is multicollinearity and why does it occur? What are its ill effects? How can it be detected? How can it be removed?

(ii) What is hazard rate? Discuss its importance in reliability theory. How is it related to conditional survival probability? State the conditions under which Weibull distribution may be used as a failure time distribution and obtain the hazard rate of this distribution.

(iii) Develop the SPRT for testing $H_0(\theta = \theta_0)$ against $H_1(\theta \neq \theta_0)$ $(\theta_1 > \theta_0)$ based on a random sample of size normal from a population following $\exp(1/\theta)$.

5) Answer any 2 (two) of the following. $10 \times 2 = 20$

(i) Define Hotelling's T^2 statistic and describe situations where it can be applied. Compare T^2 with its univariate counterpart. Prove that T^2 is invariant with respect to scale transformation.

(ii) Give the complete analysis of a 2^4 completely confounded factorial design where the highest order interaction is confounded.

(iii) Discuss the two sample Kolmogorov Smirnov statistic for testing equality of two distributions.

6) Answer any 2 (two) of the following. $10 \times 2 = 20$

(i) Derive the distribution of waiting time in system in M/M/1 queues.

(ii) Show that if $\{N_i(t), t \geq 0\}$ are independent Poisson processes with rate λ_i, $i=1,2$, then $\{N(t), t \geq 0\}$ is a Poisson process with rate $\lambda_1+\lambda_2$ where $N(t) = N_1(t) + N_2(t)$.

(iii) Consider a Markov chain with states 0,1,2,3,4 . Suppose that $P_{0,4} = 1$; and suppose that when the system is in state i, $i>0$, the next state is equally likely to be any one of the states $0,1,..,i-1$. Find the limiting probabilities of the Markov chain.

7) Derive the optimal decision rule for the deterministic demand inventory model with following specifications – production is instantaneous, total demand R during period T arises at constant rate r, unit holding cost per unit time is C_1, shortages are met by back ordering at unit cost C_2 per unit time, set up cost is C_3 per set up.

$20 \times 1 = 20$

8) Give the complete analysis of one way ANOVA with two missing observations.

$20 \times 1 = 20$

9) Explain the different schemes for explaining the oscillations in a stationary time series. Explain the use of correlogram for determining the above schemes.

$20 \times 1 = 20$

10) Explain different types of survivorship curves and discuss probability modeling of the them.

$20 \times 1 = 20$